BULLETIN OF APPLIED MECHANICS 4(15), 104-108 (2008)

Control of Translational Motion of Flexible Bodies by "Wave Method"

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Abstract— **The motivation for this work is the motion control of flexible mechanical systems, specifically the position control of the shifting parts without the residual vibrations. These systems are for example long robot arms, gantry cranes, some parts of machine tools etc..**
The actuators are coupled to the continuum in one point or in its vicinity. The other boundaries are "free". The goal of the actuators is to position the "important point" of the continuum at the other end through the flexible system and to suppress the vibrations during the motion including the complete elimination of the residual vibrations of the important point at the end of the motion.
The wave-based control method for 1D continuum has been developed [1]. This paper shows how to extend this 1D method [1] to 2D continuum. The paper shows how to compute and use the "launched" and "reflected" waves for control. If some strategy is used, the flexible system will be translated to the required position with no residual vibrations.

Index Terms— **Flexible mechanical system; residual vibrations; lumped model; mechanical waves; position control; wave-based control; launch wave; reflected wave.**

I. INTRODUCTION

This paper describes the extension of the „wave method" [1] of motion control of flexible bodies from 1D continuum to 2D continuum of 2D objects.

The control of motion of flexible bodies can be based on launched and reflected waves and these waves can be measured in the connection between the actuator and the continuum. By means of the measurement of reflected wave and its subsequent usage as launched wave it is possible to eliminate the motion of waves in the continuum, i.e. bring all points into resting state.

A 2D body is connected to two actuators where one actuator controls the motion in x direction and the other in y direction. By this means a 2D body can be generally moved in plane.

The presented paper is organized as follows. First the wave-based control for 1D is briefly summarized. Then its extension to 2D is described. Finally several open problems are mentioned.

II. "WAVES" IN A SYSTEM WITH LUMPED MASSES

This system is composed as a chain of spring-mass-spring-mass… The controlled position of the actuator is marked as $x_0(t)$. $x_n(t)$ denotes the position of the *n*-th mass in the chain. For the description of the control method and the explanation of the notion of mechanical wave exactly this model is used because of simplicity and prevention of unpredicted phenomena. Similarly this model can be imagined as FEM model of a bar with possible deformation only in one (axial) direction.

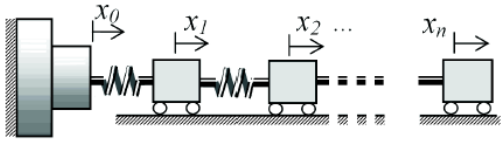

Fig. 1 Model of 1D system with lumped masses

The dynamics of the system from Fig. 1 can be modeled by classical approach, i.e. by *n* equations of motion, or alternatively by wave model. The wave model includes connected blocks with transfer functions G(s) and suitable boundary conditions. The transfer function G(s) is derived just for one motion component and supposes infinite uniform chain of masses (infinite uniform 1D lumped mass-spring system). However, the derived transfer function *G(s)* is then used for mass-spring system just with *n* masses.

The mass system from Fig. 1 can be thus modeled according to Fig. 2. It is an example of the wave model.

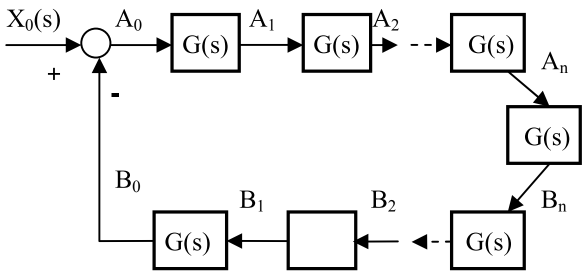

Fig. 2 Wave model of mass system with *n* DOFs

The upper branch of the diagram models the "wave" passing right that leaves the actuator in the direction into the system. This branch is depicted by letter *A*, i.e. $A_i(s)$ or $a_i(t)$. The lower branch of the diagram models the reflected wave passing back through the system in the direction to the actuator. This is depicted as $B_i(s)$ or in time domain as $b_i(t)$. The deflection $X_i(s)$ is obtained as superposition of both "waves", thus according to [2,4]

$$X_i(s) = A_i(s) + B_i(s) \tag{1}$$

The direction of arrows in the diagram represents the energy flow. The values $a_i(t)$ and $b_i(t)$ physically do not exist. It does exist only their sum that is equal to the position of certain mass $x_i(t)=a_i(t)+b_i(t)$. Therefore it is necessary to compute them by some means from measured real values that are the real motions of the masses. Thus it is necessary to derive expressions that enable to compute the above considered waves. It can be written

$$X_0(s) = A_0(s) + B_0(s) \quad (2)$$

$$X_1(s) = A_1(s) + B_1(s) \quad (3)$$

$$A_1 = GA_0 \quad (4)$$

$$B_0 = GB_1 \quad (5)$$

$$A_0 = X_0 - G[X_1 - GA_0] \quad (6)$$

$$B_0 = G[X_1 - GA_0] = X_0 - A_0 \quad (7)$$

For determination of the launched wave a_0 and reflected wave b_0 it is necessary to know the position of the actuator x_0 and the position of the first mass in the chain x_1.

Schematic representation how to compute the waves a_0 and b_0 is in Fig. 3.

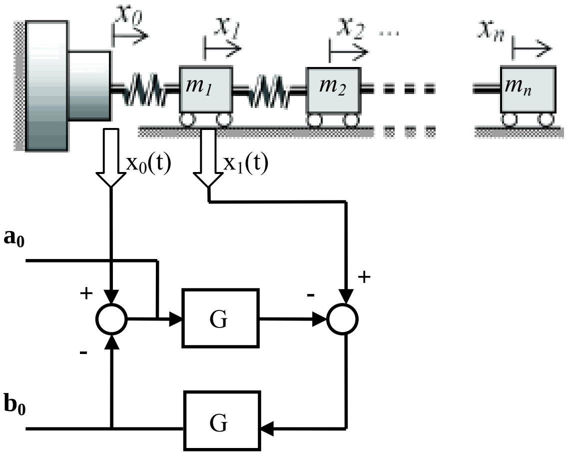

Fig. 3 Scheme of computation of waves a_0 and b_0

III. TIME DOMAIN MODEL OF ANALOGY OF TRANSFER FUNCTION G

According to [1, 4] it is possible to described the transfer function G as a transfer function (it follows from the Fig. 4, but it is the transfer function of one cell in Fig. 2) with the input of input variable p_{in} producing the output variable p_{out}

$$p_{out}(s) = G\, p_{in}(s) \quad (8)$$

The damping constant is not necessary to be selected completely accurately, the coefficient c can be chosen from the interval (0.5; 1). The mass m_1 is not necessary to be known completely precisely, it is necessary just to determine the motion p_{out} according to the Fig. 4.

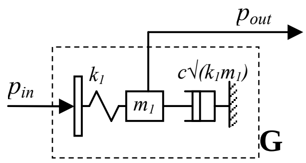

Fig. 4 Scheme of model of the transfer function G

If the above described findings are summarized it follows that for the computation of the launched and reflected waves in the place of the actuator it is necessary to know the values of the stiffness k_1, the mass of the first mass m_1 and to know the positions x_0 and x_1 (the position of the actuator and the position of the first mass m_1). It is not necessary to know the values of other masses or how many masses are in the chain. The control can work without the identification of the mechanical system, it is necessary to know just the small part around the actuator.

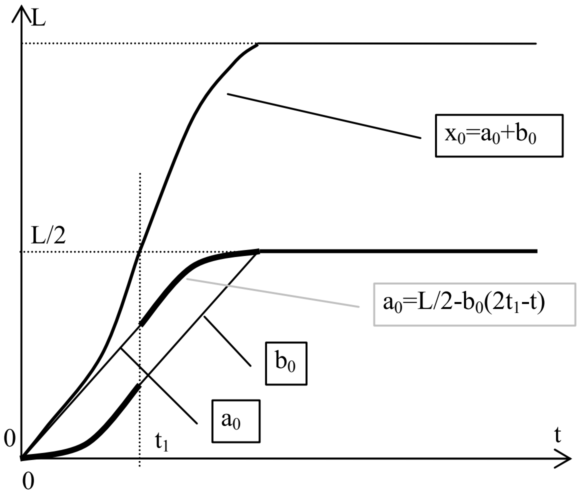

Fig. 5 Time behaviour of mechanical waves for the translation of 1D continuum by the distance L

IV. CONTROL STRATEGY

The control strategy means the way how to apply the computed waves $a_0(t)$ and $b_0(t)$ for the control of translation of the given 1D continuum. Let us suppose to move the continuum by the distance L. The control strategy describes that the position of the actuator $x_0(t)$ is controlled in two phases (Fig. 5). The first phase is for $x_0(t) \leq L/2$, the second phase is if the position of the actuator is behind the half of the required motion, i.e. $x_0(t) > L/2$.

In the first phase the control strategy is that the position $x_0(t)$ is controlled in such way that the launched wave $a_0(t)$ has the ramp shape. Generally the wave $a_0(t)$ is a linear function of time $a_0(t) = K \cdot t$ within a necessary time interval $<0, t_1>$ for the motion of the actuator $x_0(t)$ up to the position $L/2$. Simultaneously it is necessary to record the time behaviour of the reflected wave $b_0(t)$, i.e. the wave $b_0(t)$ is computed according to the scheme with the transfer functions G in the Fig. 3 within the time interval .

In the second phase the recorded time behaviour is used. It is inverted and time reversed time behaviour $b_0(t)$ from the first phase of the time interval $<0, t_1>$. It is used as the continuation of the launched wave $a_0(t)$ within the next time interval $< t_1, 2t_1>$ (Fig. 5) [3].

V. APPLICATION OF WAVE METHOD IN 2D CONTINUUM

It is supposed that in the case of linear continuum the principle of superposition enables to apply the wave method. However, the motion is decomposed into translational and rotational.

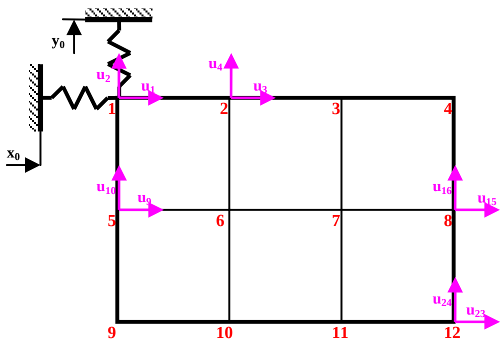

Fig. 6 The FEM model of investigated 2D continuum

VI. TRANSLATION IN 2D CONTINUUM

It is considered a motion of planar rectangular plate with the ratio of sides 3:2 (Fig. 6). The plate is modeled by FEM model. The plate is connected with actuators through stiffnesses, one in the direction X, the other in the direction y.

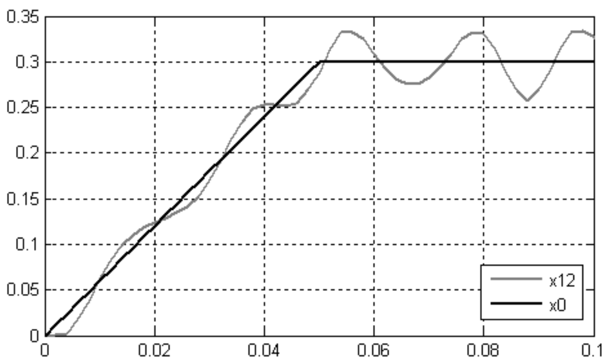

Fig. 7 Translation of the plate (axis x) without the wave control

The wave-based control theory is applied to this plate for both motion directions simultaneously and in such way that the plate rotation is prevented. Therefore the resulting force from both actuators passes through the centre of mass of the plate. Thus the required displacement in the direction y is equal -2/3 of the required displacement in the direction x

$$y_{konc} = -2/3 \cdot x_{konc} \tag{9}$$

On this kind of 2D continuum the wave method is applied for both direction functions. The results are evident from the following graphs where the time behavior is depicted for actuator and the point 12 that is the point on the other side of the continuum to be moved. Fig. 7 and Fig. 8 demonstrate the residual vibrations if the motion of the actuator in the directions x and y is carried out without the wave control. Fig. 9 and Fig. 10 demonstrate the same motion with wave control.

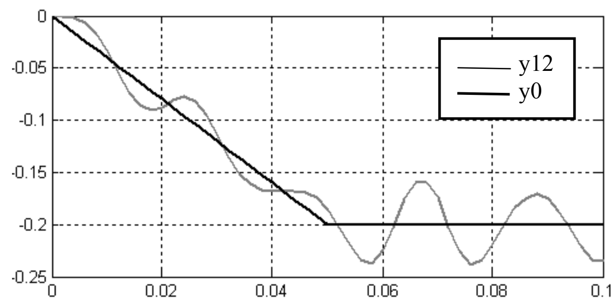

Fig. 8 Translation of the plate (axis y) without the wave control

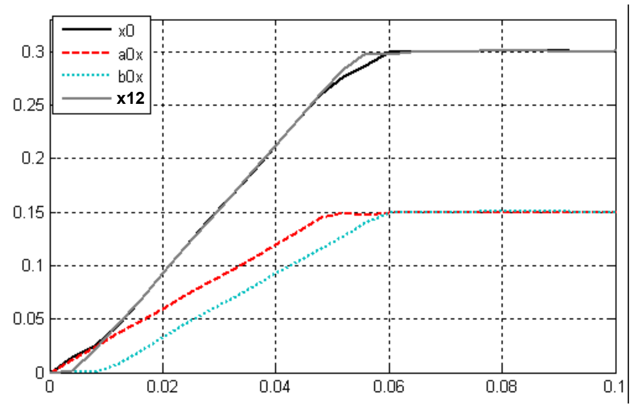

Fig. 9 Translation of the plate with wave control – axis x (x0 = x-position of the actuator, a0x = launched wave in x, b0x = reflected wave in x, x12 = x-position of the node 12)

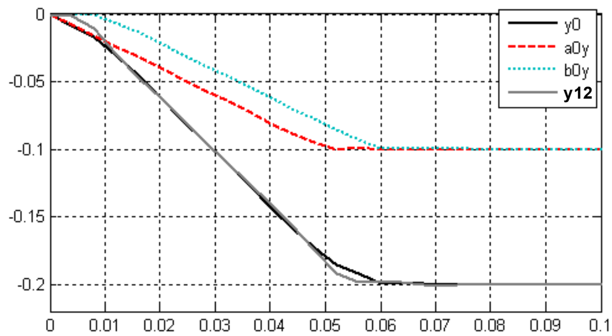

Fig. 10 Translation of the plate with wave control – axis y (y0 = y-position of the actuator, a0y = launched wave in y, b0y = reflected wave in y, y12 = y-position of the node 12)

VII. PURE ROTATION OF 2D CONTINUUM

The rotation of the plate is achieved by the attachment of rotational actuator to the plate through the torsional spring in analogy with the attachment of translational actuators. The force action of the torsional actuator is generally a moment and its influence is modeled as force action in the nodes 2 and 5 (Fig. 11). The acturators for x and y displacement remained connected. For the investigation of pure rotation they are not used but later they are necessary for general displacement in plane.

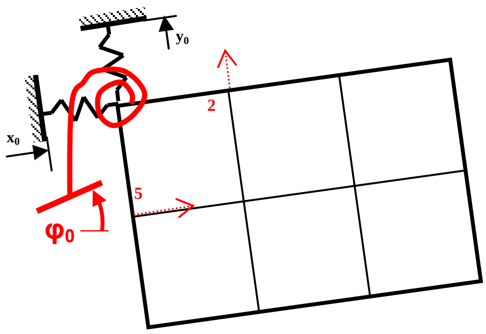

Fig. 11 Torsional actuator for plate rotation

The wave control can be applied for the pure rotation of the plate through this actuator. The same wave control has been applied for the torsional actuator. The results of the rotational motion by -0.7 radians are in Fig. 12. The time behavior of rotational waves is analogical with the time behavior of translational waves.

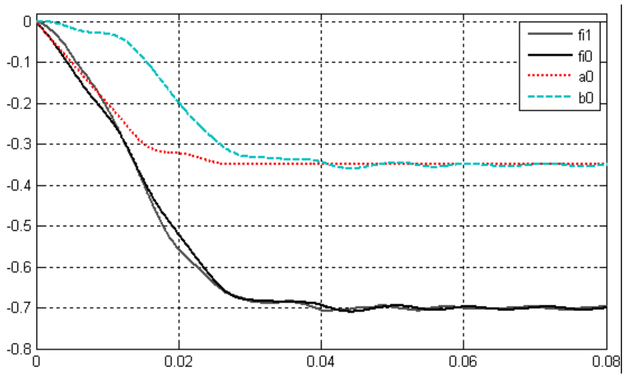

Fig. 12 Time behavior of rotational waves by pure rotation by -0.7 radians

VIII. GENERAL PLANAR MOTION

This arrangement of the plate can reach any position in the plane (x, y, φ). The control for all three motions simultaneously has been tested. The displacements x and y are not absolute but relative because the actuators X and Y rotate with the coordinate system of the plate. This is done in order the launched wave sent by the actuator X returns after the reflection back to the actuator X despite the plate is meantime rotated. This is possible during simulation instead of recomputation of the waves during the motion.

The results are not satisfactory (Fig. 13). The translations in relative coordinates x and y function correctly and the residual rotational vibrations are not caused. The rotational vibrations calm down but the plate does not reach the required angle φ, because the reflected wave $b_{0\varphi}$ settles on different (unpredicted) value than the required one.

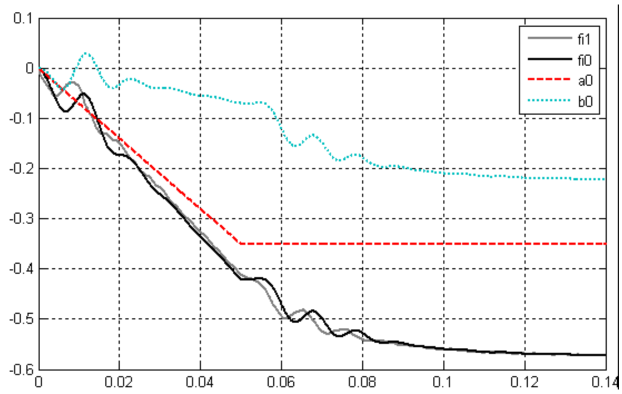

Fig. 13 Time behavior of rotational waves by simultaneous translations x=0.4, y=-0.24 and rotation by angle -0.7 rad

It has been also applied a simpler wave-based control theory that does not use the recorded reflected wave from the first phase for the second phase. Instead of that it only moves the actuator a0 as a linear function up to the value $a0=1/2 \cdot x_{required}$ and then waits [1,4].

IX. CONCLUSION

It has been demonstrated that the theory derived for 1D continuum with lumped masses can be used even for more complex (2D) continuum and not only for one direction. It has been done for the plate translational motion in general direction in plane. It also fully works for pure rotation. However, the simultaneous translation and rotation do not fully function. There is no residual vibration at the end of the motion but the rotation does not reach the required angle value. It is still an open question. Another open question is the behavior of the wave-based control on the external disturbance. If all these open question were solved then the wave-based control would be very promising control method of flexible mechanical systems consisting of a continuum.

ACKNOWLEDGMENT

The authors appreciate the support by the project “Research of new principle of mechanical and biomechanical systems with an intelligent behaviour” GACR 101/08/H068.

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