The Multiobjective Optimization of Machine Tool Construction by a Global Computation in a Whole Workspace

Jan Zavřel, Michael Valášek, Zbyněk Šika, Tomáš Vampola

***Abstract*—This paper deals with the application of newly developed methods enabling multiobjective optimization of mechanical structures by the global computations. The methods described in this paper are applied to a real mechanism. It was chosen as the kinematical structure of `TRIJOINT 900H`. Its simplified structure compounds of two bodies connected together by a rotational joint. This structure is optimized in two design parameters, a length of the left arm and a length of the right arm. These parameters are changed in the optimizing cycle and new bodies are generated. The stiffness and the accessible acceleration are computed as global criteria for every variant. The goal is to compute the Pareto set by genetic algorithms and subsequently to choose the best variant on it. The Pareto set is displayed and the solution on it is chosen finally.**

***Index Terms*—multiobjective optimization, reduction, global computation, machine tool**

I. Introduction

In general all engineering design tasks lead always to multiobjective optimization. Then the computation of the Pareto set is the basis of optimal design. Besides that the specific problem of design of mechanisms is that the performance criteria must be evaluated over the all positions in the workspace. This is computationally very demanding and using current traditional methods impossible.

Therefore a special computational tools called global computation were introduced and developed. Instead of evaluating the properties of the mechanical structure just in one position the properties are computed within similar computational time as in single position for all positions in the workspace. It has started with the global computational tools for accessible dynamics [4]. Similar method for global computation of stiffness and modal properties of mechanisms has been recently developed [7], [6].

This paper deals with the application of these new methods to the multiobjective design of a real mechanism. The two conflicting performance criteria are the stiffness and the accessible acceleration of the machine tool over the workspace evaluated in the spindle housing. The considered design variables (parameters) are the length of the left arm and a length of a right arm of the considered mechanism.

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II. Multiobjective optimization by global computation

The developed procedure for global computation of stiffness and modal properties with subsequent multiobjective optimization by genetic algorithms consists of the following steps:

1) The design variables (parameters) for the structural global computation within multiobjective optimization are selected.
2) The components (bodies) of the investigated mechanism are modeled by FEM package for several variants of the design variables.
3) The FEM models are reduced.
4) The FEM models described by mass, damping and stiffness matrices are parametrized by spline interpolation between the several variants of the design variables.
5) A grid of positions in the workspace of the mechanism is defined including the inverse kinematic transformation for the determination of position of each component (body) of the investigated mechanism in the workspace
6) The reduced FEM models are automatically interconnected in each position in the workspace.
7) The structural mechanical properties are computed for the reduced interconnected FEM model of the mechanism.
8) This computation of structural mechanical properties can be provided for any value of design variables using the spline parametrization.
9) This computation of structural mechanical properties according to the values of design variables is controlled by the multiobjective optimization using genetic algorithms.

These steps are further described in more details.

III. Optimized structure

The investigated mechanism is the `TRIJOINT 900H` machine tool. Its mechanical structure was optimized earlier yet, but by another procedures. The resulting design of bodies of the real machine structure was given at the beginning. The goal was to compute the Pareto set and to demonstrate the optimal design of the machine by depicting its mechanical properties in the design space (hopefully lying on the Pareto set).

The scheme of the kinematical structure of the main mechanism of `TRIJOINT 900H` is in Fig. III. It is a planar mechanism with 2 DOFs that is carrying out the translational motions in x and y of the spindle V. The mechanism is actuated by the drives translating the carriages A and C. The 3D FEM

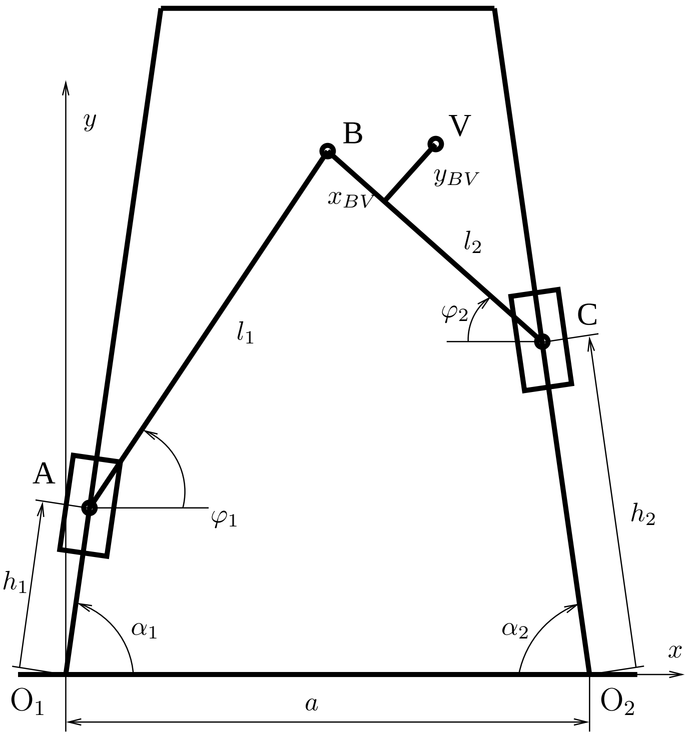

Fig. 1. The scheme of the kinematical structure of TRIJOINT 900H.

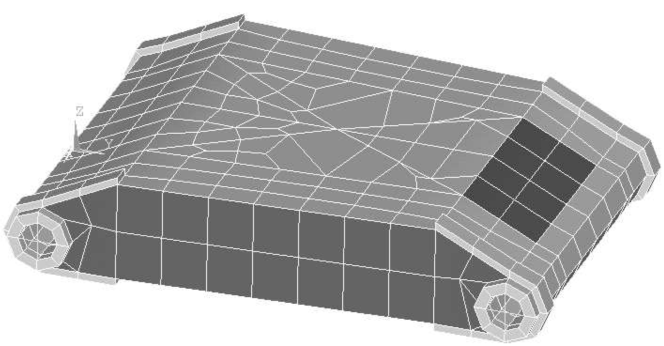

Fig. 2. The body of the left arm.

models of the body of the left arm and of the right arm are in the figures Fig. 2 and Fig. 3. They are modelled in the ANSYS environment. The structure is reduced into the planar problem.

Both bodies of the arms modeled in the 3D space are projected to two-dimensional bodies (Fig. III and Fig. III) and they are modeled in ANSYS by 2D finite elements. There are used finite elements with two nodal coordinates in every node, especially degrees of freedom in *x* and *y* axes (PLANE42 element).

They are modeled as a front view of the 3D models. Both bodies are divided in three parts. Two parts are on sides, there are joints with bearings and the spindle housing. These side parts reamins during the optimization procedure always the same. The middle part is modified in agreement with the optimized parameters.

Both bodies will be generated later along the optimizing design parameters proposed in the optimizing cycle. The bodies must be generated in several dimension variants from this reason. These variants are used for computing the interpolation functions which are used for generating the bodies with new dimensions.

A. Preparation of the bodies

Both bodies are modeled in the ANSYS environment and than exported to the ASCII text files. All bodies are than imported to the MATLAB environment, where all following modifications and computations are made.

The local coordinate systems at both bodies are introduced. These local coordinate systems are important for the further transformations of the bodies, especially rotations.

The joints are prepared in the first step, where the artificial middle joints are used [7]. The code numbers of the nodes are in the next step renumbered so that all numbers of both bodies will not be in conflict.

The models of the bodies are than ready for reduction and application of boundary conditions, forces and torques.

The optimization process uses thousands of variants. An optimization process (computation) with original bodies would be a very time-consuming task. Therefore instead of original model the reduced models are used so that the computational time is decreased.

There are several methods how to reduce mechanical structural models [6]. An iterated IRS method [1] and [2] and a static reduction method are used. The iterated IRS method is used for models which are used for computations of modal characteristics and the static reduction for computations of deformations.

B. Body reduction and changing the dimensions

The reduced models of the bodies are used for faster optimization because the description of the original system leads to very large matrices. This system with two bodies is described by the mass and stiffness matrices. Every matrix has a several thousands rows and columns (degrees of freedom) and computations with such matrices are not efficient.

After careful analysis the iterated IRS reduction method was selected [7] for model reduction. Firstly, the master and slave degrees of freedom must be selected. The master degrees of freedom will be kept and the slave ones will be omitted. As the master degrees of freedom that are kept, the nodes (every

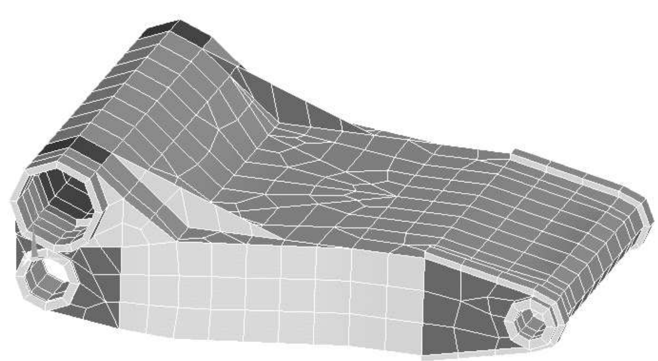

Fig. 3. The body of the right arm.

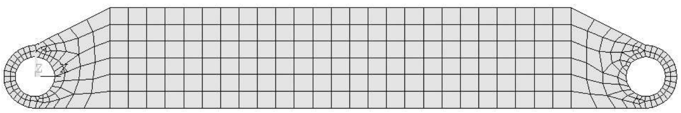

Fig. 4. The left body (simplified).

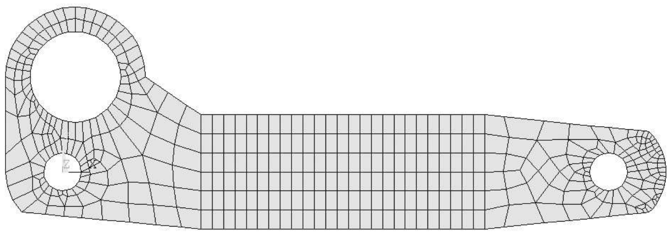

Fig. 5. The right body (simplified).

node has a couple of degrees of freedom) that are important are selected. The important nodes are the joint nodes, the nodes needed for the reduction, the nodes with applied loads and the nodes that are important for the computation of the performance criteria (e.g. a spindle housing).

The system is described by the equation of motion

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}, \tag{1}$$

which is divided into master $\mathbf{x}_m$ and slave $\mathbf{x}_s$ parts

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_m \\ \ddot{\mathbf{x}}_s \end{bmatrix} + \\ \begin{bmatrix} + & \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ & \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{x}_m \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{f}_m \\ \mathbf{f}_s \end{bmatrix}. \tag{2}$$

The goal is to determine the suitable transformation $\mathbf{x} = \mathbf{T}\mathbf{x}_m$ between the original coordinates and the selected master coordinates. From the dynamic reduction [3] the transformation matrix $\mathbf{T}_D$ becomes

$$\mathbf{T}_D = \begin{bmatrix} \mathbf{I} \\ \left[\mathbf{K}_{ss} - \mathbf{M}_{ss}\omega^2\right]^{-1} \left[\mathbf{K}_{sm} - \mathbf{M}_{sm}\omega^2\right] \end{bmatrix}. \tag{3}$$

From the description (2) and using (3) the slave part of the system can be expressed as

$$\mathbf{x}_s = -\mathbf{K}_{ss} \left[\mathbf{I} - \omega^2\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\right]^{-1} \left[\mathbf{K}_{sm} - \mathbf{M}_{sm}\omega^2\right] \mathbf{x}_m. \tag{4}$$

Using a binomial theorem

$$(1 \pm x)^n = 1 \pm \binom{n}{1} x \pm \binom{n}{2} x^2 \pm O(4) \tag{5}$$

yields

$$\begin{aligned} &\mathbf{I} - \omega^2\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1} \binom{-1}{1} + \mathbf{O}(\omega^4) = \\ &= \mathbf{I} + \omega^2\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1} + \mathbf{O}(\omega^4). \end{aligned} \tag{6}$$

Using (4) it is derived

$$\begin{aligned} \mathbf{x}_s &= -\mathbf{K}_{ss}^{-1} \left[\mathbf{I} + \omega^2\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\right] \left[\mathbf{K}_{sm} - \mathbf{M}_{sm}\omega^2\right] + \\ &+ \mathbf{O}(\omega^4)]\mathbf{x}_m \\ &= -\mathbf{K}_{ss}^{-1} \left[\mathbf{K}_{sm} - \omega^2\mathbf{M}_{sm} + \omega^2\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} - \right. \\ &\left. - \omega^4\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\mathbf{M}_{sm} + \mathbf{O}(\omega^4)\right] \mathbf{x}_m, \end{aligned} \tag{7}$$

and by neglecting $\mathbf{O}(\omega^4)$ and all higher components the slave part of the system is described as follows

$$\mathbf{x}_s = -\mathbf{K}_{ss}^{-1} \left[\mathbf{K}_{sm} + \omega^2 \left(\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} - \mathbf{M}_{sm}\right)\right] \mathbf{x}_m. \tag{8}$$

The solution of the eigenmodes of the system reduced by a static reduction gives the harmonic solution as (9).

$$\begin{aligned} \tilde{\mathbf{x}}_m &= \mathbf{x}_m \sin(\omega t) \\ \dot{\tilde{\mathbf{x}}}_m &= \mathbf{x}_m\omega \cos(\omega t) \\ \ddot{\tilde{\mathbf{x}}}_m &= -\mathbf{x}_m\omega^2 \sin(\omega t). \end{aligned} \tag{9}$$

The reduced system will be described by the equation

$$\mathbf{M}_G\ddot{\mathbf{x}}_m + \mathbf{K}_G\mathbf{x}_m = \mathbf{0} \tag{10}$$

and substituting from the (9)

$$-\mathbf{M}_G\omega^2\mathbf{x}_m + \mathbf{K}_G\mathbf{x}_m = \mathbf{0}, \tag{11}$$

it gives

$$\omega^2\mathbf{x}_m = \mathbf{M}_G^{-1}\mathbf{K}_G\mathbf{x}_m \tag{12}$$

After the next substitution to the (8) the slave coordinates $\mathbf{x}_s$ are then given by

$$\begin{aligned} \mathbf{x}_s = &\left[-\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} + \mathbf{K}_{ss}^{-1} \left(\mathbf{M}_{sm} - \right.\right. \\ &\left.\left. -\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm}\right) \mathbf{M}_G^{-1}\mathbf{K}_G\right] \mathbf{x}_m. \end{aligned} \tag{13}$$

The complete expression is then in the matrix form

$$\begin{aligned} &\begin{bmatrix} \mathbf{x}_m \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} \end{bmatrix} \mathbf{x}_m + \\ &\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{ss}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \\ &\begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} \end{bmatrix} \mathbf{M}_G^{-1}\mathbf{K}_G\mathbf{x}_m. \end{aligned} \tag{14}$$

And finally, after all necessary substitutions the final transformation to the reduced system by the IRS method is

$$\mathbf{T}_{IRS} = \mathbf{T}_S + \mathbf{SMT}_S\mathbf{M}_G^{-1}\mathbf{K}_G, \tag{15}$$

where the $\mathbf{S}$ matrix is

$$\mathbf{S} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{ss}^{-1} \end{bmatrix}. \quad (16)$$

The reduced system is obtained by the transformion

$$\begin{aligned} \mathbf{K}_{IRS} &= \mathbf{T}_{IRS}^T \mathbf{K} \mathbf{T}_{IRS} \\ \mathbf{M}_{IRS} &= \mathbf{T}_{IRS}^T \mathbf{M} \mathbf{T}_{IRS} \end{aligned} \quad (17)$$

The iterated IRS method is then obtained by the iterated procedure (Frisewell et al. 1998). The first reduced system is computed by the static reduction and then the reduction is iterated

$$\mathbf{T}_{IRS,i+1} = \mathbf{T}_S + \mathbf{S}\mathbf{M}\mathbf{T}_{IRS,i}\mathbf{M}_{IRS,i}^{-1}\mathbf{K}_{IRS,i} \quad (18)$$

By the iterated procedure the reduced system is more accurate.

The choice of master coordinates as the nodes on the body boundary gives a very good agreement [6] for models before and after reduction. In our case the MAC criterion (Modal Assurance Criterion) for the original and the reduced 2D body of the left arm is in the Fig. 6. The reduction degree (the ratio between the number of original and reduced degrees of freedom) is 90 %. There are only 20 modal shapes shown. The very good agreement between the original and the reduced

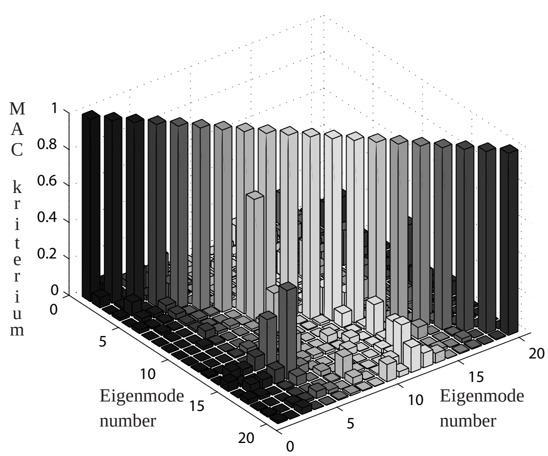

Fig. 6. The MAC criterion of the left body for one configuration.

C. Connection of the bodies

Both bodies must be connected together, they are connected by the rotational joint which is modeled by the artificial central node [5], [6] in each joint (Fig. 7). In the middle of the joint of the body there is created the artificial node which is linked to the nodes on the body by the defined stiffness.

All bodies must be transformed to the desired position before connecting all bodies together. This position is given by the position in the workspace. All bodies are connected by the defined link together after the transformation. This link has the desired stiffness. The structure is connected and prepared to compute the performance criterion. Two performance criteria are considered - the stiffness of the machine tool evaluated in the spindle housing and the accessible acceleration both over the whole workspace. An example of the MAC criterion of the connected bodies in one position in the workspace is in the Fig. 8.

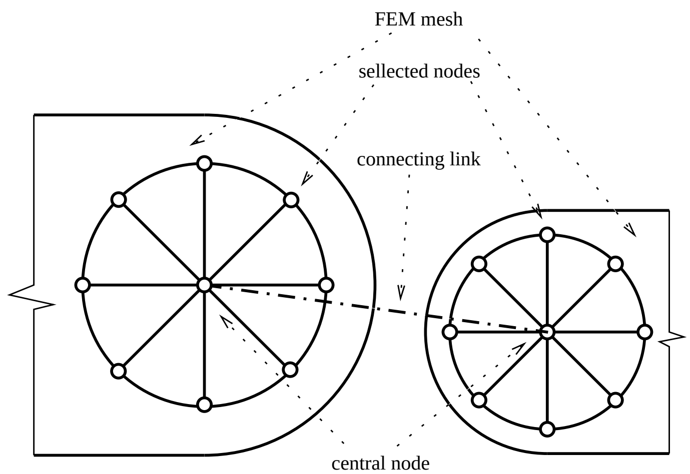

Fig. 7. The rotational joint with an artificial central node.

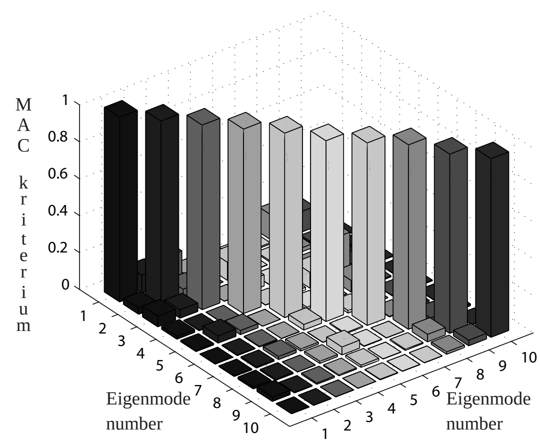

Fig. 8. The MAC criterion of the connected reduced bodies in one position in the workspace.

IV. MULTIOBJECTIVE OPTIMIZATION

The multiobjective optimization process goes from the proposal of the design variables (parameters) and generation of new bodies through the body transformations and their connections to the evaluation of the criteria and the analysis of the set of all possible solutions. The particular steps of the optimization loop are in the figure Fig. 9. The goal is to determine the design parameters of the admissible designed variants on the Pareto set by the optimization. The admissible design variant means in our case the reachability of all positions in the workspace. The design variables (parameters) are the lengths of the arms. The considered performance criteria are the stiffness of the machine tool evaluated in the spindle housing and the accessible acceleration both evaluated over the whole workspace. The generated and evaluated design

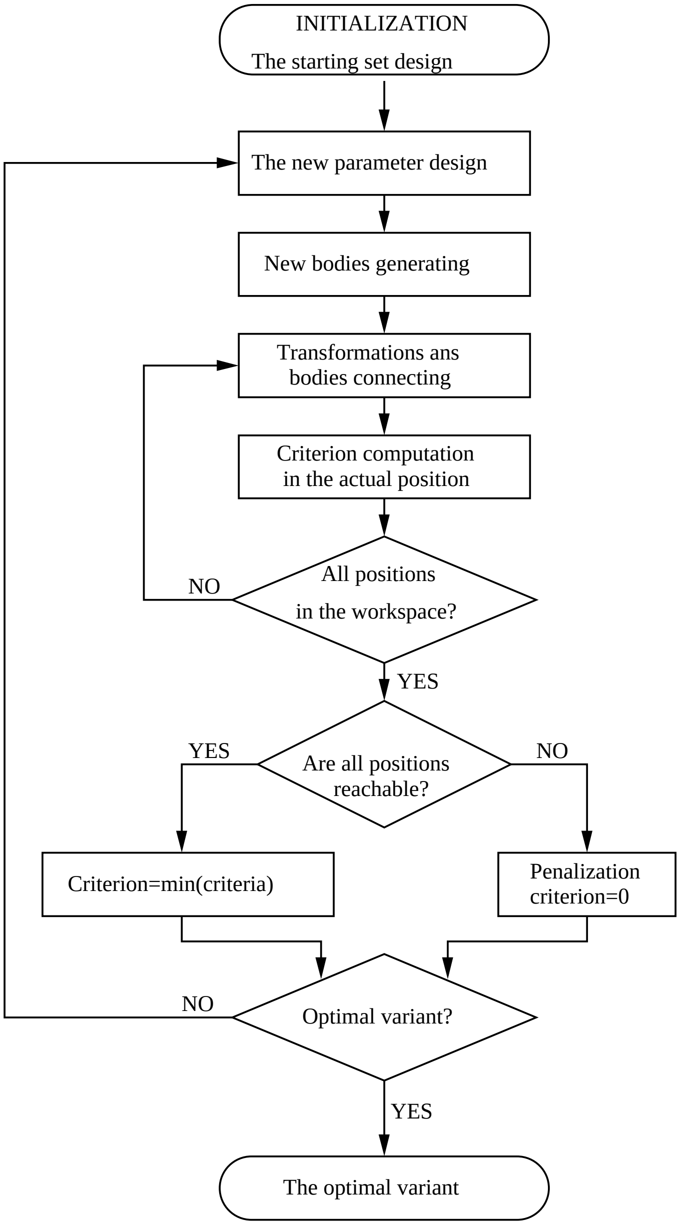

Fig. 9. The optimization procedure.

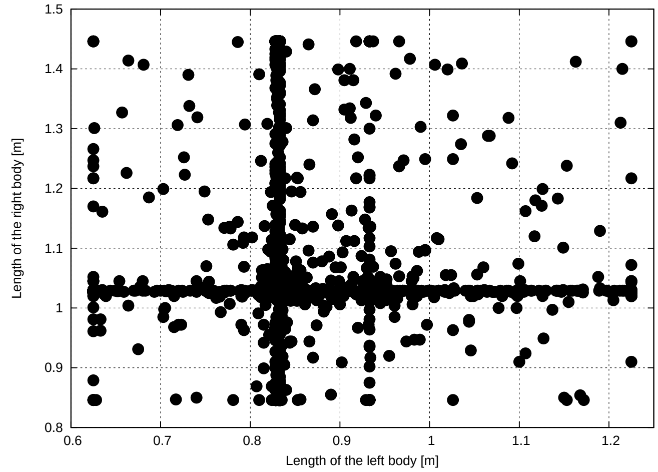

Fig. 10. The set of generated design variants (optimization steps).

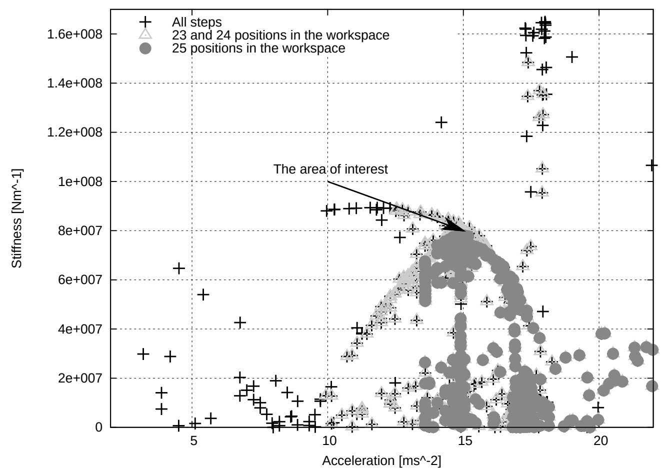

Fig. 11. Pareto set for stiffness – acceleration.

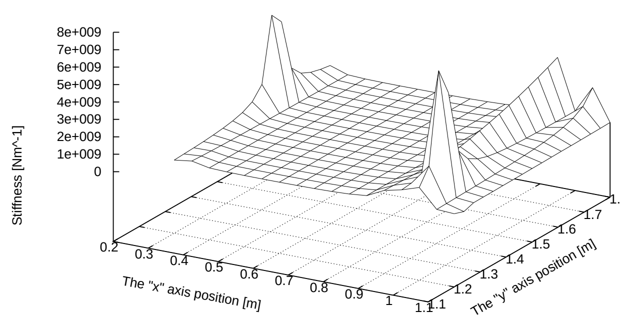

Fig. 12. TRIJOINT - the global map of stiffness for one of the variants.

variants are in the figure Fig. 10. The resulting Pareto set as the dependence stiffness – acceleration is in the figure Fig. 11.

The aim is to reach all positions in the whole workspace and to achieve the best criteria. The workspace is divided by the mesh of 5 times 5 points, overall 25 positions. The set of optimization steps in the figure Fig.11 is divided into three parts. There are markers ”cross” in the first part that are all variants during the optimization. The second part marked by round points are the variants that reach all 25 positions in the workspace. The third part (triangle markers) consists of all variants where only 23 and 24 positions in the workspace could be reached, i.e. the choice of the design parameters prevent the mechanism to cover the whole workspace.

The Pareto-set changes when the criteria changes only a little. It is the most distinguished property between the ”filled circle” markers and ”triangle” markers sets. A very small change of the request of the performance criteria allows us change the behaviour of the system radically.

In the middle of the figure the Pareto-set forms a very important area. It includes the most interesting design variants. Especially important part of the Pareto-set is its part where the break in the slope of Pareto-set occurs (the arrow points at). The small increase of the dynamics results into the rapid decrease of the stiffness. It is the place for the choice of the optimal design variant. The design parameters of `TRIJOINT 900H` were selected exactly here. The newly reconstructed Pareto-set passed through this optimal design point. This proves the validity of the developed global computation and optimization method. The original computation of the properties of `TRIJOINT 900H` were performed on significantly more simplified computational models. However, the developed global computation method for stiffness and modal properties enables to compute the mechanical properties by more advanced and complex models in comparable computational time as by the previous methods.

As a side effect the efficient computation of the stiffness map of `TRIJOINT 900H` in the whole workspace is possible. An example of such resulting stiffness map is in Fig.12.

V. Conclusion

The newly developed efficient global computation method of structural mechanical properties and its usage for the multiobjective structural optimization of mechanisms (movable mechanical structures) using genetic algorithms were described. This method was applied for the multiobjective optimization of the mechanical structure of the `TRIJOINT 900H` machine tool. The Pareto-set of multiobjective optimization of stiffness and accessible acceleration over the whole workspace was computed and compared with the original design. The original design variant of `TRIJOINT 900H` underlies on this Pareto-set. This proves the validity of the developed method and the optimality of the originally performed design. Such optimization process usually takes from days to weeks, but using the described methods the computational time is cut down just to hours.

The developed computational tools are very powerful. The described example of machine tool optimal design has demonstrated that the resulting optimal variants can be very sensitive to the required properties. The designers must be very careful when specifying them and then considering the resulting variants.

Acknowledgements

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