Synthesis and Implementation of Vibration Suppression by 6 DOF Active Platform

Svatoš P., Šika Z., Steinbauer P., Zavřel J., Vampola T., Valášek M.

***Abstract*—The paper deals with the design of the mathematical model of active platform with six degrees of freedom. For the purpose of micro-positioning and vibration suppression, the controller synthesis for active platform with piezoactuators is presented via robust H_∞ control method implemented in HIFOO, which ensures the fixed-order controller. The finite element method model of this active platform is reduced and transferred into a suitable state-space model form with analysis of the choice of sensors for real application and performance improvement. Main targets are design of platform model, synthesis of controller with fixed order, and experimental validation of control strategy.**

***Index Terms*— Fixed-order Controller Design, HIFOO, H_∞ control, FEM modelling, Dynamic Model, Modal Analysis.**

I. INTRODUCTION

MODERN robust control strategies [1, 2] with the possibility of integration of sensors and actuators into machine structure bring the attractive idea of smart structures [3, 4]. Active components influence behaviour of machine in order to improve its properties like positioning or vibration suppression. Moreover, rigid bodies are only idealization and flexibility should be taken into account. Most common is usage of finite element method (FEM). These FEM models are accurate, approaching the reality, but they are not usually suitable for control requirements due to computational complexity. Thus a low-order structural model is desired for control design.

Our motivation:

For improvement of mechanism positioning accuracy and elimination of vibration, we enhanced the planar parallel experimental machine Sliding Star [5] by second active platform structure (Fig. 1). The second active platform with 6 degrees of freedom (DOF) consists of additional platform and 6 piezoactuators. The motion/positioning of machine platforms is controlled hierarchically. The large motion of Sliding Star's primary platform with 3 DOF is controlled redundantly by four electrical drives. The small relative motion of secondary active platform, which respects the large motion of primary platform, causes precise positioning and vibration suppression. Secondary platform could be used as the stabilized end-effector for e.g. spindle positioning.

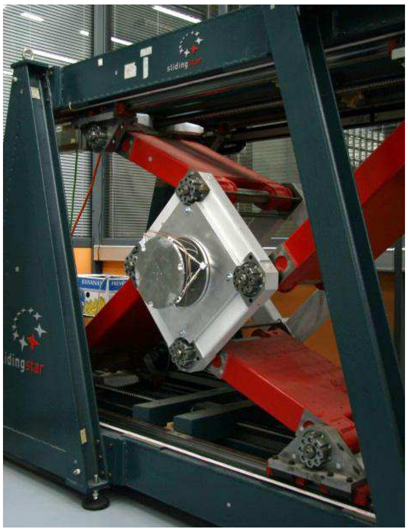

Fig. 1 Sliding Star with active platform

This paper is focused on the synthesis of H_∞ tracking fixed-order controller for the active platform with 6 DOF for purpose of vibration suppression together with tracking. In order to design a controller, the FEM model was reduced and converted into modal coordinates and generalized state-space model form [6] (required to H_∞ controller synthesis) with regard to sensors and input and output values. The generalized state-space model of this active platform is derived. H_∞ controller synthesis [6] is supplied by HIFOO [7, 8], the optimization package for MATLAB.

The paper is further organized into these sections. Section 2 illustrates the active platform, its frame and real components.

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Ing. Petr Svatoš: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Petr.Svatos@fs.cvut.cz)

Prof. Ing. Zbyněk Šika, Ph.D.: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Zbynek.Sika@fs.cvut.cz)

Ing. Pavel Steinbauer, Ph.D.: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Pavel.Steinbauer@fs.cvut.cz)

Ing. Jan Zavřel, Ph.D.: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Jan.Zavrel@fs.cvut.cz)

Doc. Dr. Ing. Tomáš Vampola: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Tomas.Vampola@fs.cvut.cz)

Prof. Ing. Michael Valášek, DrSc.: Czech Technical University in Prague, Faculty of Mechanical Engineering, Technická 4, Praha 6, 166 07, Czech Republic, (Michael.Valasek@fs.cvut.cz)

The following section 3 explains system modelling and choice of sensor, inputs and outputs. Section 4 describes the local H_∞ control design via HIFOO optimization. Section 5 contains simulation results. Section 6 presents experimental validation. Overall results are summarized in section conclusions.

II. Active Platform with 6 DOF

The active platform consists of two bases which are connected by 6 piezoactuators. The concept is inspired by the Stewart platform with 6 active legs, well known as Hexapod [9]. Configuration of 6 piezoactuators is orthogonal and allows 6 DOF. This solution in the form of cube configuration should provide an excellent controllability to satisfy the requirements of micro-positioning and vibration suppression. The photo of real active platform is shown in the Fig. 2.


Fig. 2 The active platform photo

The bottom base includes connection point for connection with the Sliding Star. Each piezoactuator is connected to basis through the adapter with flexible tip. Following Fig. 3 illustrates components of platform.


Fig. 3 Components of platform – flexible tip (on the left) and piezoactuator

Each of piezoactuators is controlled independently via voltage through amplifier. There is also a possibility to measure the total extension length.

III. The Model of The Platform

The first step was to have an accurate platform model. Next steps were to make model reduction (static reduction and reduction considering modal properties). Finally a generalized state-space model with sensors choice was composed in order to catch robust controller design.

A. FEM Model of Platform

The FEM model of the platform with 6 DOF according to the real realization (as one can see in Fig. 2) was created using ANSYS. The next figures (Fig. 4 and Fig. 5) illustrate this model. The first reduction - the static reduction, was carried out by ANSYS and thus the FEM model of the platform was reduced to 3000 DOF.

We obtained equations of the linear mechanical system

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{1}$$

where mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ are being used in following section on modal modelling.

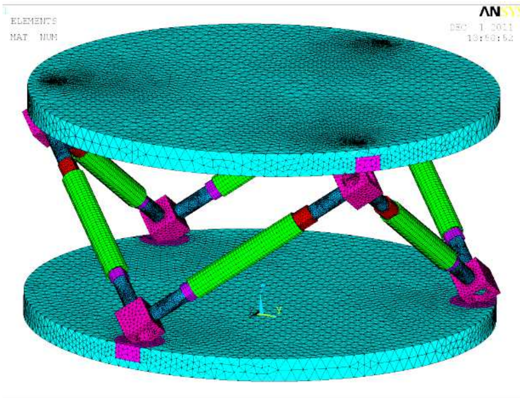
Fig. 4 Platform FEM model grid

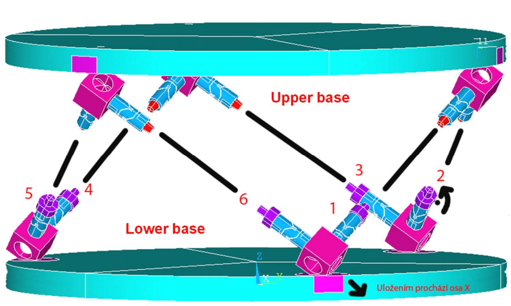

Fig. 5 Platform FEM model account

The FEM model respects free motion of platform, free rigid body modes.

For the case when the platform is fixed to the other machine or to the base frame we obtained

$$\mathbf{M}_f \cdot \ddot{\mathbf{x}} + \mathbf{B}_f \cdot \dot{\mathbf{x}} + \mathbf{K}_f \cdot \mathbf{x} = \mathbf{F_d} + \mathbf{M_{rig}} \cdot \ddot{\mathbf{x}}_\mathbf{p} + \mathbf{K_{rig}} \cdot \mathbf{x_p} + \mathbf{F_a} \tag{2}$$

where $\mathbf{M_{rig}} \cdot \ddot{\mathbf{x}}_\mathbf{p} + \mathbf{K_{rig}} \cdot \mathbf{x_p}$ are forces expressing forced motion and $\ddot{\mathbf{x}}_\mathbf{p}, \mathbf{x_p}$ are new inputs – acceleration and position of base frame.

B. Modal State-Space Form of System

The equation (1) was transferred by modal transformation $\mathbf{x} = \mathbf{V}\mathbf{q}$ and left multiplication by modal matrix $\mathbf{V}^T$ into well-known equation

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{q}}+\mathbf{V}^T\mathbf{B}\mathbf{V}\dot{\mathbf{q}}+\mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{q}=\mathbf{V}^T\mathbf{f}\,. \tag{3}$$

The proportional damping and mass matrix normalization are considered.

The modal form of the state space model can be derived in different variants [10] Let us consider the form in which the state vector of considered modal description has the special form of $\mathbf{z}=[\dots \quad \mathbf{z}_i \quad \dots]^T$, where $\mathbf{z}_i=[\Omega_i q_{mi}, \dot{q}_{mi}]^T$, q_{mi} and $\dot{q}_{mi}$ are modal elastic coordinates and modal elastic velocities.

The second model reduction, in modal coordinates level, was done by singular perturbation approximation (SPA) [11, 12] with respect to residual modes of system for different outputs (position, velocity or acceleration).

By using these procedures the state space form

$$\begin{bmatrix}\dot{\mathbf{z}}\\ \mathbf{y}\end{bmatrix}=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\begin{bmatrix}\mathbf{z}\\ \mathbf{u}\end{bmatrix} \tag{4}$$

was derived.

C. Generalized State-Space Mode of System

For the needs of H_∞ controller synthesis, let's consider the generalized state-space system [6], which represents an open-loop plant *P* as follows:

$$\begin{bmatrix}\dot{\mathbf{x}}\\ \mathbf{z}\\ \mathbf{y}\end{bmatrix}=\begin{bmatrix}\mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2\\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12}\\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{D}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{w}\\ \mathbf{u}\end{bmatrix}. \tag{5}$$

where $\mathbf{A}\in\Re^{n\,x\,n}$, $\mathbf{D}_{12}\in\Re^{z\,x\,m}$ and $\mathbf{D}_{21}\in\Re^{p\,x\,w}$, $\mathbf{x}$ is state, $\mathbf{w}$ is external disturbance input and command, $\mathbf{u}$ is control input (actuators), $\mathbf{z}$ is performance output (to be control) and $\mathbf{y}$ is measured output (sensors).

This form allows splitting of system inputs into actions and disturbances or commands, and similarly system outputs into sensors and performance outputs. The generalized form is asked for H_∞ control synthesis. The following Fig. 6 explains the generalized model of plant with tracking, filters and limitation of control input magnitude. Filters, which are used in performance output, causes stress certain frequency area and thus influence a controller design through the criterion of optimality.

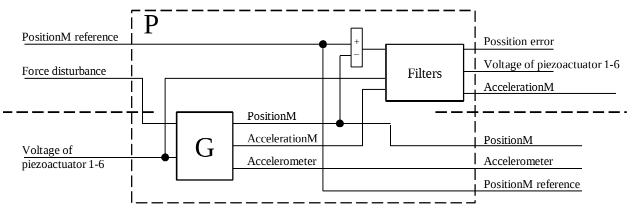

Fig. 6 Generalized plant *P*

The external disturbance and input, actuators, performance outputs and sensors choice for considered platform are illustrated in the Fig. 7 as follows:

- force disturbance $\mathbf{F_d}$ and forced motion have an effect on the active platform,
- six actuators are control input,
- position in x-axis of upper base of active platform is controlled together with vibration suppression and limitation of control input magnitude,
- acceleration in three points A_1, A_2, A_3 with position in x-axis point *P* of upper base of active platform are measured (there is possibility together with six total extension of actuators).

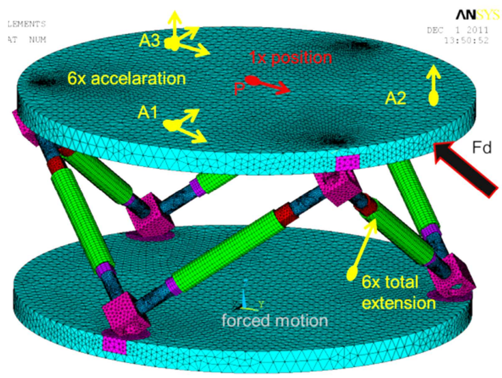

Fig. 7 The choice of inputs and outputs for generalized plant

Equations (6) – (9) express this choice as well.

$$\mathbf{w}=[\,\mathbf{x_{refP}}\;\; \mathbf{w_d}\,]' \tag{6}$$
$$\mathbf{u}=[\mathbf{U_{p1}}\;\mathbf{U_{p2}}\;\mathbf{U_{p3}}\;\;\mathbf{U_{p4}}\;\mathbf{U_{p5}}\;\;\mathbf{U_{p6}}\,]' \tag{7}$$
$$\mathbf{z}=[\,\mathbf{errorx_P}\;\;\mathbf{a_{Px}}\;\mathbf{a_{Py}}\;\mathbf{a_{Pz}}\;\;\mathbf{U_{p1}}\;\mathbf{U_{p2}}\;\mathbf{U_{p3}}\;\;\mathbf{U_{p4}}\;\mathbf{U_{p5}}\;\;\mathbf{U_{p6}}\,]' \tag{8}$$
$$\mathbf{y}=[\mathbf{a_{A1x}}\;\;\mathbf{a_{A1y}}\;\mathbf{a_{A2z}}\;\mathbf{a_{A3x}}\;\mathbf{a_{A3y}}\;\mathbf{a_{A3z}}\;\;\mathbf{x_P}\;\;\mathbf{x_{refP}}\;\;\mathbf{\Delta_{1-6}}]' \tag{9}$$

Filters are suitable designed low pass for position error, high pass for limitation of control input and band pass for acceleration.

C. Model properties

Computed properties of model of platform are summarized in the following table and figure. The first seven nonzero eigenfrequencies are shown in the table. First six eigenfrequencies for platform as free body equal zero.

Eigenfrequencies of platform

NONZERO EIGEN-FREQUENCY [Hz]	Fixed to base frame	Free Body
1.	148.9	369.9
2.	149.7	381.2
3.	261.3	473.7
4.	377.1	475.1
5.	386.6	545.5
6.	389.6	852.3
7.	842.0	856.8

Fig. 8 presents transfer function from force disturbance $\mathbf{F_d}$ to x-axis acceleration of point *P*.

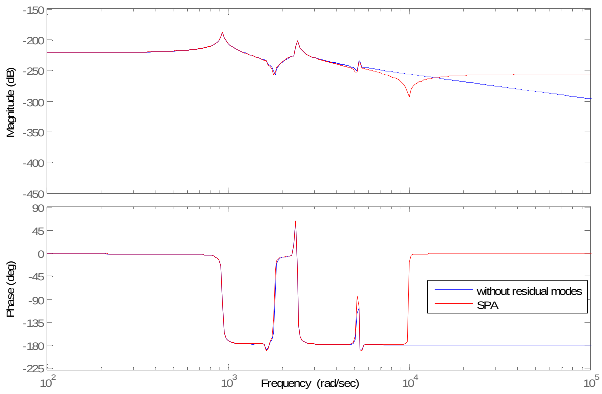

Fig. 8 Transfer function – Fixed to base frame

IV. H$_\infty$ Controller Synthesis

The open-loop generalized plant is controlled by another linear system, called the controller [6]. The final closed-loop feedback system is shown in Fig. 9. The task is to minimalize the H$_\infty$ norm from input ***w*** to output ***z***.

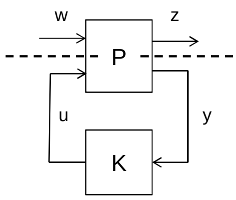

Fig. 9 Closed-loop system

This task has been done by optimization method HIFOO [7, 8], which is based on local approach H$_\infty$ controller. The target was to design the controller matrixes elements $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$, $\hat{\mathbf{D}}$.

HIFOO is a powerful and efficient local optimization method for robust fixed-order controller design, which is implemented in MATLAB. It can be profitably used for vibration suppression of flexible systems. The direct result of the HIFOO optimization is the fixed-order controller. The algorithm has two phases: *stabilization* (the maximum of the real parts of the closed-loop system make negative) and *optimization* (searching for a local minimizer). HIFOO is very flexible and robust tool, which overcomes the nonsmooth and nonconvex function. It is able to converge on the stabilizing controller even from bad unstable guesses.

V. Simulations and Results

The dynamic generalized model of the active platform with 6 DOF was introduced in the previous sections. The aims of simulations were the design of a robust fixed-order controller for tracking and vibration suppression of platform fixed to the base frame.

Structural model:

Matrix order of generalized state-space model of structure:

$$\mathbf{A} = [20\ x\ 20],\quad \mathbf{B}_1 = [20\ x\ 2],\quad \mathbf{B}_2 = [20\ x\ 6],$$
$$\mathbf{C}_1 = [10\ x\ 20],\quad \mathbf{C}_2 = [8\ x\ 20],\quad \mathbf{D} = [18\ x\ 8]$$

Note that matrix **D** is nonzero.

Fixed-order regulator:

The 3$^{\text{th}}$ order controller matrixes were designed. The number of randomly generated and added guesses by HIFOO is ten. The other options were set to default.

The following table shows the simulation results. The magnitude of the H$_\infty$ norm, maximum of real parts of eigenvalues and tracking response were analysed.

	HIFOO and GAOT
$\lVert H_\infty \rVert$	0.2966
max(Re)	-14.004

THE RESULTS FOR HIFOO OPTIMIZATION METHOD

In the Fig. 10, there is presented a transfer function from force disturbance $\mathbf{F_d}$ to x-axis position of point *P*.

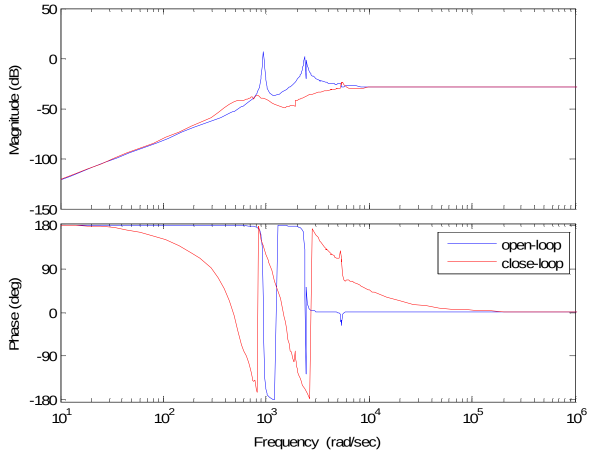

Fig. 10 Transfer function from disturbance to upper base of platform

Next figures show a time history of the step response (Fig. 11) and Dirac impulse response (Fig. 12) as disturbance force.

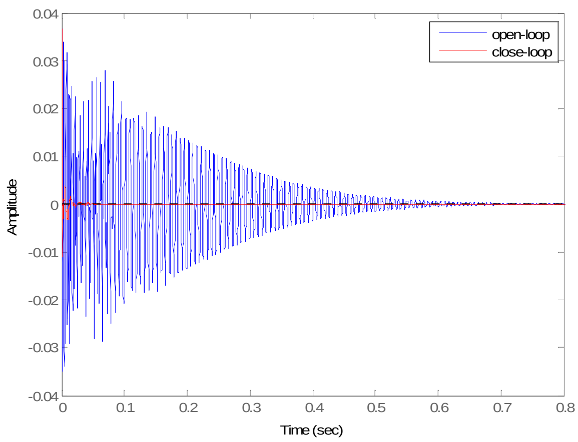

Fig. 11 Step response – from disturbance to acceleration $\mathbf{x_P}$

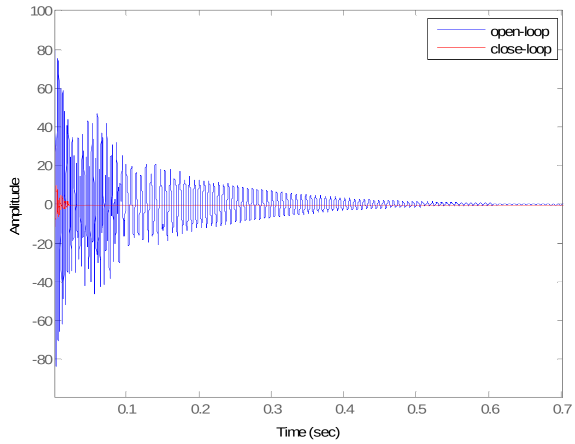

Fig. 12 Dirac impulse – from disturbance to acceleration $\mathbf{x_p}$

Obtained results in these figures indicate that the controller suppresses the vibration. Amplitude becomes significantly smaller and achieves the constant values faster. The final closed-loop system is also stable.

The following Fig. 13 illustrates the tracking ability as step response from reference signal to position of upper base of the active platform.

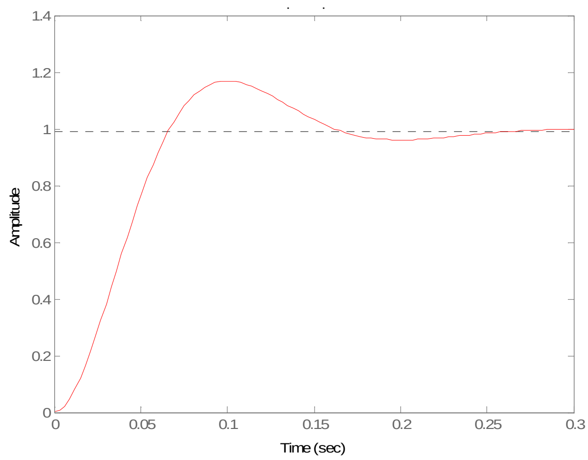

Fig. 13 Step response – from reference position to position $\mathbf{x_p}$

VI. Experimental validation

The experimental modal analyses (EMA) have been done for validation of reduced model. Fig. 14 and Fig. 15 show the first eigenfrequency for modelling of platform fixed in the base frame and as a free body. These results indicate very good correspondency EMA with FEM and reduced generalized model (see table Eigenfrequencies of platform).

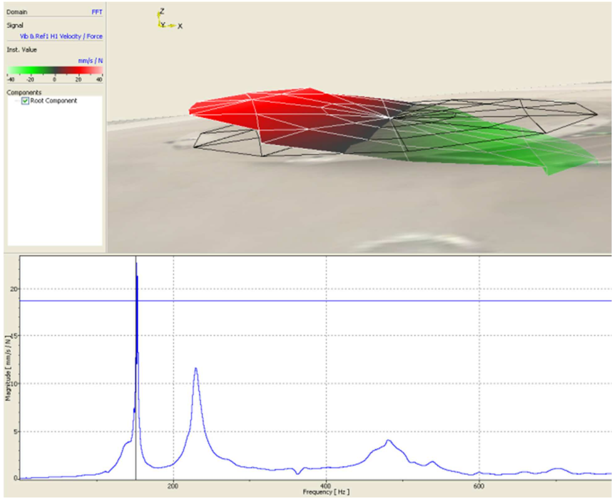
Fig. 14 Modal analysis – Fixed to base frame -151.7Hz

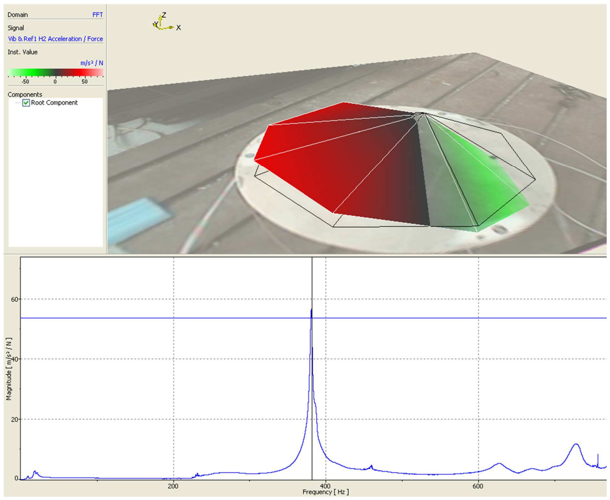
Fig. 15 Modal analysis – Free body - 381.5Hz

Next work, which was realized in 2011, is experiments with control of one-level active truss structure in order to vibration suppression. This one-level active truss structure was equipped with accelerometers and additional absolute distance measurement via laser interferometer or relative absolute rate measurement via laser vibrometer. This experiment is illustrated in the Fig 16, where the drifted platform was enhanced by additional mass.


Fig. 16 Active truss structure with accelerometers and additional mass

There are shown the results of this experiment in the next Fig. 17.

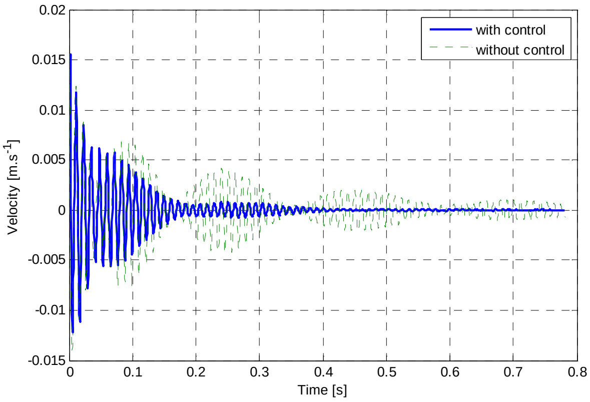

Fig. 17 Experimental results of an active vibration suppression

VII. CONCLUSION

The generalized state-space model of active platform with 6 DOF has been presented and the tracking fixed-order robust H_∞ controller has been designed. The model of platform was successfully reduced and transferred into a generalized state-space form for controller synthesis needs. The H_∞ controller with lower order than controlled system order was successfully designed by the HIFOO optimization method. The results show that the controller suppresses the vibration and is able to follow the reference position signal.

The EMA provides the same model properties. The experiment with control for vibration suppression was done with one-level active truss structure, which was equipped with accelerometers.

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