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Springforward phenomenon of angular sections of composite materials – analytical, numerical and experimental approach

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Abstract— **Residual stresses, which are set in the fiber reinforced composites during the laminate curing in a closed form, lead to dimensional changes of composites after extracting from the form and cooling. One of these dimensional changes is called "springforward" (also "spring-in" or "springback") of angle sections. Other dimensional changes are warpage of flat sections of composite or displacement of single layers of composite for example. In our case L-section from C/PPS composite with a symmetrical lay-up was analysed. An analytical model which covers temperature changes, a chemical shrinkage during curing and a moisture change was used. There were also a FEM analysis, for predicting the springforward, and experimental analysis carried out.**

Index Terms— **Composite structures, Experimental analysis, FEM analysis, Springforward phenomenon**

I. INTRODUCTION

CHANGE in composite dimensions is related to many parameters as: part angles, thicknesses, lay-ups, flange length, but also tool materials, tool surface or cure cycles [1]. When the composite L (or U) section is extracted from the form that was cooled to the room temperature, the change in the angle of the part (Fig.1) can be observed.

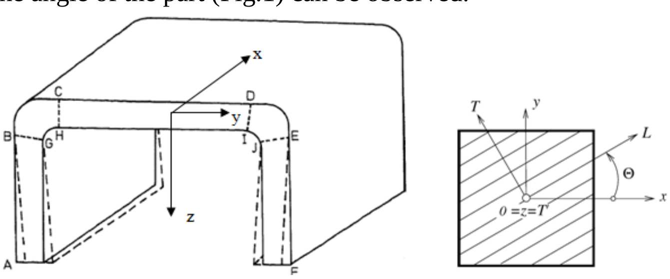

Fig.1. Distortion of moulded U-section [4]

Tool angles have to be modified to affect this problem. The tool design is based on either the "rules of thumb", from the past experience, or on the trial-and-error. For the angular parts, the compensation is normally between 1 and 2,5°. The most common problem found, using a standard factor, is that the springforward may vary with the lay-up, material, cure temperature, etc. Therefore, what worked once does not necessarily work next time [2]. The main cause of springforward was the mismatch in thermal expansion along and across the fibers in a laminate [3]. But there are also other causes which will be discussed in the next analysis.

II. SPRINGFORWARD OF UNIDIRECTIONAL COMPOSITE PLATE

A. Derivation of springforward equation owing to temperature

The springforward (SF) of an orthotropic plate of thickness h with unidirectional fiber (under angle Θ) can be defined and derived according to [5]. In Fig.1, it can be seen a coordinate system and in Fig.2, the signification of side lengths.

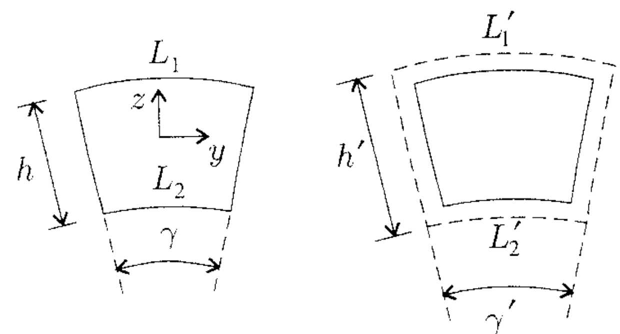

Fig.2. Derivation of springforward effect [5]

Let´s define springforward as a relative change in the angle of the upper and lower surfaces of the plate element. The plate has different lengths which causes the different strains due to the temperature

$$SF = \frac{\gamma' - \gamma}{\gamma}. \tag{1}$$

The angle γ is in the corner radius of composite plate and it can be computed as the length changes of surfaces divided by the shell thickness h

$$\gamma = \frac{L_1 - L_2}{h}, \tag{2}$$

and the angle γ' due to heating is computed similarly

$$\gamma' = \frac{L_1' - L_2'}{h'}. \tag{3}$$

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The relative elongation due to the change in temperature is determined by a well known equation depending on the coefficient of thermal expansion and temperature difference $\varepsilon^t = \alpha\Delta T$. In the case of unidirectional composite its longitudinal thermal expansion is different than that in the transversal direction. Therefore, also the temperature deformations are different in the longitudinal and transverse directions. They can be computed as

$$\varepsilon_L^t = \alpha_L \Delta T\ , \quad \varepsilon_T^t = \alpha_T \Delta T\ , \quad \varepsilon_{T'}^t = \varepsilon_z^t = \alpha_{T'} \Delta T \qquad (4)$$

where α_L and $\alpha_T = \alpha_{T'}$ are the longitudinal and transverse coefficients of thermal expansion. Relative thermal deformations can be then transformed from the coordinate system *O(L,T)* to the rotated system *O(x,y)* (see Fig.1). The equation used for transforming deformations (transformation matrix $\boldsymbol{T}$) can be applied to obtain the coefficients of thermal expansion, That´s why

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \boldsymbol{T}^{-1} \begin{bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{bmatrix},$$

$$\boldsymbol{T} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \qquad (5)$$

where the apparent coefficient of thermal shear α_{xy} can be seen. The relative thermal deformations in coordinate system *O(x,y)* can be expressed as

$$\begin{bmatrix} \varepsilon_x^t \\ \varepsilon_y^t \\ \gamma_{xy}^t \end{bmatrix} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \Delta T. \qquad (6)$$

The arc lengths on the upper and lower surfaces of the plate and its new thickness will be, after deformation, expressed as

$$L_1' = L_1 + \alpha_y \Delta T L_1, \quad L_2' = L_2 + \alpha_y \Delta T L_2, \quad h' = h + \alpha_{T'} \Delta T h. \qquad (7)$$

Substituting Eqs.(2) – (7) into Eq.(1), the relationship for the springforward is received as

$$SF = \frac{(\alpha_y - \alpha_{T'})\Delta T}{1 + \alpha_{T'}\Delta T} \cong (\alpha_y - \alpha_{T'})\Delta T = \varepsilon_y^t - \varepsilon_z^t. \qquad (8)$$

That equation states that the springforward is approximately equal to the difference of the plate longitudinal a transverse temperature deformations. The transverse deformation is thought as the deformation through the plate thickness.

B. Springforward effect due to other influences on volume change

The springforward effect can be caused by other influences on the plate volume change in analogical way as that due to its change in temperature. The procedure described in [6] took into account not only the thermal expansion but also the volume change due to the material absorptivity. Also the structural effect of volume shrinkage during the cure cycle is taken into account. The angle change $\Delta\gamma$ equation of the moulded piece can be written as

$$\gamma = \Delta\gamma_t + \Delta\gamma_h + \Delta\gamma_c = \gamma \frac{(\alpha_y - \alpha_{T'})\Delta T}{1 + \alpha_{T'}\Delta T} + \gamma \frac{(\beta_y - \beta_{T'})\Delta c}{1 + \beta_{T'}} + \gamma \frac{(\phi_y - \phi_{T'})}{1 + \phi_{T'}}, \qquad (9)$$

which superimpose all the three effects: $\Delta\gamma_t$ is the temperature part of angle change, $\Delta\gamma_h$ is the change in angle due to the hygroscopic effect and $\Delta\gamma_c$ is change in angle due to shrinkage effect during the cure cycle.

The first term of the equation is the temperature part of angle change which is the result of residual stresses in the material during the cooling of ΔT, when the material is completely stiff. The second term describes the deformation due to the moisture change and absorptivity of composite (humidity effect). It is thought that the moisture change is

$$\Delta c = c - c_0 \qquad (10)$$

where c_0 is the initial moisture. The deformation due to the moisture change is

$$\varepsilon^h = \beta\Delta c \qquad (10)$$

where β is the coefficient of expansion which expresses the unitary moisture absorption. In the case of unidirectional composite, the longitudinal moisture expansion is different than the transversal one. Therefore the moisture deformations are also different in the longitudinal and transversal directions. They can be computed as

$$\varepsilon_L^h = \beta_L \Delta c\ , \quad \varepsilon_T^h = \beta_T \Delta c\ , \quad \varepsilon_{T'}^h = \varepsilon_z^h = \beta_{T'} \Delta c, \qquad (11)$$

likewise the transformation of thermal expansion coefficients from *O(L,T)* to *O(x,y)*, the moisture coefficients can be transformed as

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \boldsymbol{T}^{-1} \begin{bmatrix} \beta_L \\ \beta_T \\ 0 \end{bmatrix}. \qquad (12)$$

The strain caused by the moisture change is described in the system *O(x,y)* by the equation

$$\begin{bmatrix} \varepsilon_x^h \\ \varepsilon_y^h \\ \gamma_{xy}^h \end{bmatrix} = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} \Delta c. \qquad (13)$$

The third term of Eq.(9)

$$\Delta\gamma_c = \gamma \frac{(\phi_y - \phi_{T'})}{1 + \phi_{T'}} \qquad (14)$$

is caused by the resin solidification and it is related to the stresses occurred in the material during the cure cycle. Term ϕ_y is the longitudinal shrinkage and $\phi_{T'}$ is the transverse (through thickness) shrinkage during curing. This structural effect of volumetric change of matrix due to the crystallization will be important in composites with thermoplastic matrices that change their phase from amorphous to crystalline one during heating and curing. During curing and crystallization, the semi-crystalline matrices shrink due to crowding of the mass (the crystals have higher density then the amorphous phase). The shrinkage of semi-crystalline matrix due to this effect can be significantly higher than that due to the temperature change. In Fig.3, it can be seen the dependence of the change in the specific volume on the temperature, where T_g stands for the glass transition temperature, T_m for the melting temperature and T_c is the temperature of crystallization during a slow cooling of the melt [7]. It is evident that amorphous polymers haven´t got the effect of volumetric change due to the recrystallization, so the influence of temperature change is crucial.

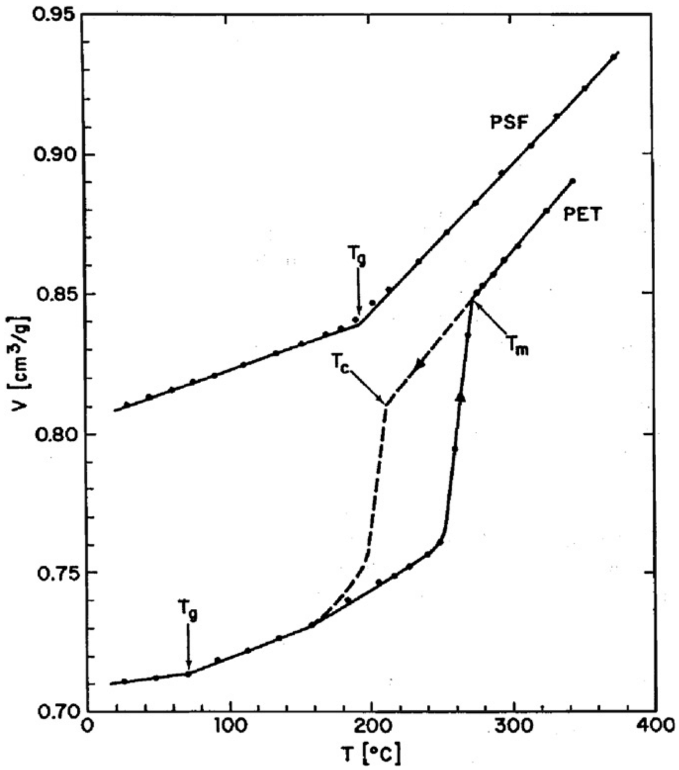

Fig.3. Specific volume of polysulfone (PSF, amorphous) and polyethylene terephtalate (PET, semicrystalic) as a function of temperature during atmospheric pressure [7]

All the three types of strain (temperature, moisture absorption and volumetric change due to recrystallization) will be marked as one generalized strain ε_y^{thc} and ε_z^{thc}.

III. SPRINGFORWARD OF LAMINATED COMPOSITE PLATE

Let´s have a laminated composite plate with i=1…N layers with thickness H ($H=\sum_i^N h_i$). According to the classical lamination theory (CLT, e.g.[5]), the strain and curvatures of the middle plane of laminated composite plate, loaded with unit forces and moments, can be obtain. Accordingly, it can be stated that strain and curvatures, arised from the temperature and moisture changes, will produce mechanical forces and moments. According to CLT

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} N_x^{thc} \\ N_y^{thc} \\ N_{xy}^{thc} \\ M_x^{thc} \\ M_y^{thc} \\ M_{xy}^{thc} \end{Bmatrix} \right) \quad (15)$$

where A_{ij}, B_{ij} and D_{ij} are the generally known elements of membrane stiffness, bending-extension coupling stiffness and bending stiffness matrices, and quantities N_i^{thc}, M_i^{thc} are defined by equations

$$N_i^{thc} = \int Q_{ij}\varepsilon_j^{thc} dz,$$
$$M_i^{thc} = \int Q_{ij}\varepsilon_j^{thc} z dz. \quad (16)$$

with the fact that integration boundaries are from *–H/2* to *H/2*. N_i^{thc} and M_i^{thc} have the same dimension as N_i and M_i and they are called the resultants of the thermohygrocrystallic unit internal forces and moments. The plane strain $\varepsilon_i^{0,thc}$ and relative change in curvature κ_i^{thc} will arise due to N_i^{thc} and M_i^{thc}at absence of N_i and M_i and they can be calculated by

$$\begin{Bmatrix} \varepsilon_x^{0,thc} \\ \varepsilon_y^{0,thc} \\ \gamma_{xy}^{0,thc} \\ \kappa_x^{thc} \\ \kappa_y^{thc} \\ \kappa_{xy}^{thc} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\ a_{12} & a_{22} & a_{26} & b_{12} & b_{22} & b_{26} \\ a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \\ b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \\ b_{12} & b_{22} & b_{26} & d_{21} & d_{22} & d_{26} \\ b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} N_x^{thc} \\ N_y^{thc} \\ N_{xy}^{thc} \\ M_x^{thc} \\ M_y^{thc} \\ M_{xy}^{thc} \end{Bmatrix}. \quad (17)$$

If the lay-up of the composite is symmetric (according to CLT there is no connection between normal forces and moments or between axial strains and curvatures) then the springforward of the complete composite can be calculated by Eq.(8) which was derived for one layer. The value of the through-thickness strain (computed in next chapter) was substituted to Eq.(8) and longitudinal strain computed from Eq.(17)

$$\varepsilon_y^{thc} = \varepsilon_y^{0,thc} = \{a_{12} \quad a_{22} \quad a_{26} \quad b_{12} \quad b_{22} \quad b_{26}\} \begin{Bmatrix} N_x^{thc} \\ N_y^{thc} \\ N_{xy}^{thc} \\ M_x^{thc} \\ M_y^{thc} \\ M_{xy}^{thc} \end{Bmatrix}. \quad (18)$$

A. Laminated composite plate with symmetrical lay up

Due to the temperature and moisture changes and the shrinkage during recrystallization, the composite layer thickness change is calculated as an integral ε_z from the bottom to the top surface of the layer

$$\Delta h = \int_{-h_b}^{h_t} \varepsilon_z dz. \quad (19)$$

When Eq.(19), which means the thickness change for one layer, is rewritten into a matrix form and extended with components describing moisture and recrystallization, the following equation is obtained

$$\varepsilon_z = \varepsilon_3 = [S_{13} \quad S_{23} \quad S_{36}] \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} + \Delta T\alpha_3 + c\beta_3 + \phi_3 \quad (20)$$

where

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}. \quad (21)$$

Transformation of this equation to that for stresses leads to

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = [\bar{Q}]\left(\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} - \Delta T\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix} - c\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{bmatrix} - \begin{bmatrix}\phi_x\\ \phi_y\\ \phi_{xy}\end{bmatrix}\right) \quad (22)$$

Strains are given by

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + z\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix}. \quad (23)$$

Then the change in thickness of the complete composite due to the change of temperature ΔT will be

$$\Delta H^t = \sum_{k=1}^{N}\left\{[S_{13} \quad S_{23} \quad S_{36}][T_q]_k\left((z_k - z_{k-1})[\bar{Q}]_k\left(\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} - \Delta T\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\right) + \frac{z_k^2 - z_{k-1}^2}{2}[\bar{Q}]_k\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix}\right) + (z_k - z_{k-1})(\Delta T(\alpha_3)_k)\right\} \quad (24)$$

where z is the coordinate of given layer. For the thickness change due to moisture, ΔT and α will be replaced with c and β. Then the thickness change will be

$$\Delta H^h = \sum_{k=1}^{N}\left\{[S_{13} \quad S_{23} \quad S_{36}][T_q]_k\left((z_k - z_{k-1})[\bar{Q}]_k\left(\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} - \Delta c\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{bmatrix}_k\right) + \frac{z_k^2 - z_{k-1}^2}{2}[\bar{Q}]_k\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix}\right) + (z_k - z_{k-1})(\Delta c(\beta_3)_k)\right\}. \quad (25)$$

Analogously with the thickness change due to the recrystallization, the parts with ΔT disappear and the symbol α will be replaced with Φ

$$\Delta H^c = \sum_{k=1}^{N}\left\{[S_{13} \quad S_{23} \quad S_{36}][T_q]_k\left((z_k - z_{k-1})[\bar{Q}]_k\left(\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} - \begin{bmatrix}\Phi_x\\ \Phi_y\\ \Phi_{xy}\end{bmatrix}_k\right) + \frac{z_k^2 - z_{k-1}^2}{2}[\bar{Q}]_k\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix}\right) + (z_k - z_{k-1})(\Phi_3)_k\right\}. \quad (26)$$

The total change in the laminate thickness will be

$$\Delta H^{thc} = \Delta H^t + \Delta H^h + \Delta H^c. \quad (27)$$

The temperature through-thickness strain will be

$$\varepsilon_z^t = \frac{\Delta H^t}{H}. \quad (28)$$

So the coefficient of thermal expansion through the plate thickness can be written

$$\alpha_z = \frac{\varepsilon_z^t}{\Delta T}. \quad (29)$$

Other coefficients can be computed in a similar way.

IV. ANALYTICAL CALCULATION OF SPRINGFORWARD

In our case, the carbon composite with PPS matrix was analyzed. PPS is semi-crystalline hi-tech plastic which is used mainly for aircraft applications. The crucial data for calculating the springforward are the coefficients of shrinkage Φ. The linear shrinkage of fiber reinforced PPS due to cooling is 0,2-0,5% [8], while it is about 1,2-1,5% with the pure PPS without fibers.

The elastic constants, coefficients of thermal expansion and coefficients of shrinkage valid for the fibers (f) and matrix (m) applied, can be seen in Table 1 (the estimated values according to [9,10] are denoted with *).

Table 1. Thermoelastic properties of fiber and matrix

E_{fL}[MPa]	**E_{fT}[MPa]**	**ν_f[-]**	**G_{fLT}[MPa]**
230000	15000	0,3	50000
$G_{fTT'}$[MPa]*	**α_{fL}[C^{-1}]**	**α_{fT}[C^{-1}] ***	**Φ_f[%]**
27000	-3,8.10^{-7}	12,5.10^{-6}	0
E_m[MPa]	**ν_m[-]**	**α_m[°C^{-1}]**	**Φ_m[%]**
3800	0,36	5,2.10^{-5}	1,5

The fiber volume V_f is 49%. The thermoelastic characteristic and coefficients of shrinkage of unidirectional lamina are obtained using 3D micromechanics equations from [9]. The results can be seen in Table 2.

Table 2. Thermoelastic properties for unidirectional lamina

E_L[MPa]	**E_T[MPa]**	**$E_{T'}$[MPa]**
114638	7961	7961
G_{LT}[MPa]	**$G_{TT'}$[MPa]**	**$G_{LT'}$[MPa]**
4372	4155	4372
ν_{LT}[-]	**$\nu_{TT'}$[-]**	**$\nu_{LT'}$[-]**
0,3306	0,916	0,3306
α_L[C^{-1}]	**α_T[C^{-1}]**	**$\alpha_{T'}$[C^{-1}]**
5,055.10^{-7}	2,44.10^{-5}	2,44.10^{-5}
Φ_L[%]	**Φ_T[%]**	**$\Phi_{T'}$[%]**
2,536.10^{-4}	0,4526	0,4526

At the end, the coefficients of thermal expansion and coefficients of shrinkage for the complete C/PPS composite with lay-up of $[[(0,90)/(\pm 45)]_4/(0,90)]_s$ can be obtained according to the chapter 4.1. and are listed in Table 3

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Table 3. Coefficients of thermal expansion and coefficients of shrinkage for the complete composite

α_x[C^{-1}]	α_y[C^{-1}]	α_{xy}[C^{-1}]	α_z[C^{-1}]
2,49.10^{-6}	2,49.10^{-6}	0	4,36.10^{-5}
Φ_x[-]	Φ_y[-]	Φ_{xy}[-]	Φ_z[-]
3,77.10^{-4}	3,77.10^{-4}	0	0,0087

When these values are substituted into Eq.(9), the change in temperature and initial angle of curvature are known and the zero change in moisture is expected, we obtain the result. For the angle of 90° and the temperature change of 160°, we obtain the change in angle of 1,17° (which is 1,3%). The relationship between the springforward and the change in temperature can be seen in Fig.4.

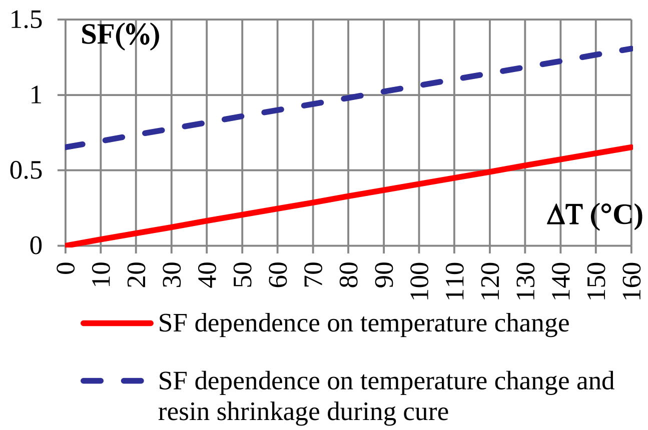

Fig.4. Relationship between the springforward (with and without calculating the resin shrinkage) and the change in temperature

V. EXPERIMENT

The main goal of the measuring was to assess the strain of carbon composite with thermoplastic matrix due to a temperature change. The devices used for the measuring were:

- Temperature sensor: PT100, CRZ Platinium Thin Film Element
- Contactless infrared thermometer FLUKE 574
- Laser profilometer ScanControl LLT 2800-25
- Optical distance measurement CHRocodile M4

A Description of the experiment

The specimen was heated up about 100° C in an oven and then cooled down to the room temperature (25-27° C). The measurement starts mainly at the temperature of 95° C and ends at 35° C after 30 min. Seven measurements in four modes were carried out. The sampling rate of the laser was 20 Hz and optical distance measurements 32 Hz. Relative displacements between the points scanned by laser were evaluated as angular displacements of the specimen vertical part in each time step. Then the springback values were calculated and compared with the analytical and numerical solution, see Fig.5.

B Interpretation of measured data

The accuracy of measuring device is in µm which means that the measurement took into account also the surface roughness. This accuracy makes the evaluation more complicated because of the volume data measured and also the fact that the measured points are changing with the deformation of the specimen. Therefore, the results taken are actually average values or regression curves. The angular displacements are measured as the springforward of L-profile but also as its volumetric change due to temperature. These two physical effects are difficult to separate. Another point to discussion is the fact that we have no information about temperature inside the specimen (temperature information is taken just from the surface).

VI. NUMERICAL CALCULATION OF SPRINGFORWARD

A finite element (FE) model was created and analyzed for thermo-elastic strains using ABAQUS solver. The results of FE analysis is conducted to be compared with the analytical springforward model as well as the experimental results. As described in the analytical procedure, the springforward angle is the angle difference between the original mould angle and the angle of moulded part, divided by the value of original mould angle. An agreement between the predicted springforward angle from the analytical model and the finite element analysis was examined to define the inputs parameters, which creates a practical tool for the engineers and designers in composite manufacturing.

The linear elastic material constants were verified on the bone specimen and compared with the experimental data. The FE model and the strain field results of the composite L section, after cooling about ΔT=60° C and with lay-up: $[[(0,90)/(\pm 45)]_4/(0,90)]_s$, is shown in Fig. 4. The model was solved with the hexahedron incompatible mode elements and two types of material properties. The material orthotropic linear elastic and thermal expansion coefficients, for the definition of one unidirectional lamina of composite section, are presented in Table 2, while for the second solutions are listed in Table 3. The theoretical material constants, applied in the tables, are from the two different sources of material linear elastic properties available [11], [12].

- AIMS 05-09-002 - $[(0,90)]_{3s}$, E(Warp/Weft) = 58/58±3 GPa
- VZLU experimental report - $[0,(0,45)_2]_s$, E = 37 GPa, ν = 0.272 in tensile test direction.

VIII. RESULT COMPARISON AND CONCLUSIONS

A derivation of the springforward phenomenon for the unidirectional composite plate, and also for the laminated composite plate with symmetrical lay-up, both influenced with temperature, moisture and the volumetric change during recrystallization, is described in this paper. This method was used for analytical calculation of given laminated C/PPS plate.

The springforward of this plate was experimentally measured and the measurement results were compared with the analytical ones with a good agreement. Numerical model was created and analyzed for thermo-elastic strains using ABAQUS solver.

The comparison of analytical, experimental and FEM simulation results for the corner section of C/PPS composite is presented in Fig.5. The results show a large range of experimental data and very good agreement of the analytical solution with the FE results of material model with the experimental linear elastic constants and orthotropic thermal expansion constants for the complete composite, Tab.3. Moreover, the results of the FE material model, which were compared with the coefficients solved by the classical lamination theory (Tab.2), are conservative.

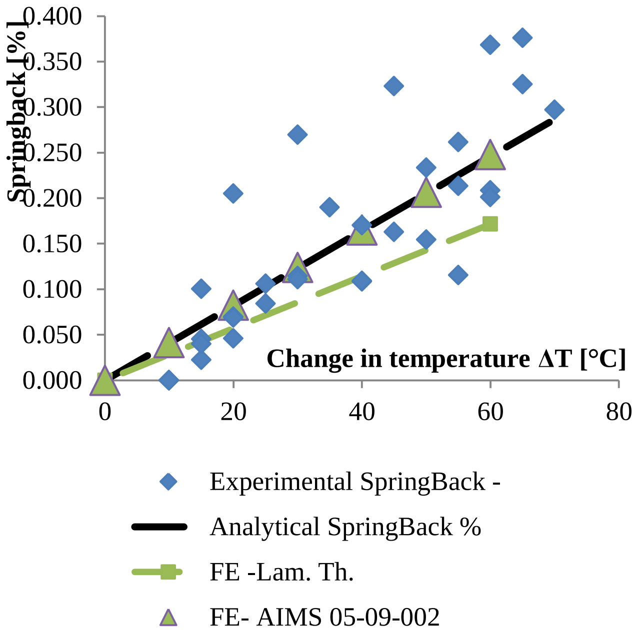

Fig.5. Comparison of analytical, numerical and experimental springforward data

Future works will be focused more on the spingforward phenomenon of single and double curvature plates with a general lay-up and also on such experiments, where also the springforward caused by recrystallization (using higher temperature range, controlled cooling, etc.) can be measured.

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