Mullins Effect in Aorta and Limiting Extensibility Evolution

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***Abstract*—Cyclic uniaxial tensile tests with samples of human aorta were performed with an aim to obtain data describing the Mullins effect of arterial tissue. Due to presumed anisotropy of an aorta, reported widely, both samples oriented longitudinally and circumferentially were tested in each case. Every tested sample underwent cyclic tension limited to a certain value of a stretch four times, consecutively the limit of sustained deformation was increased and subsequent four cycles were performed. Significant stress softening of aortic tissue and residual strains were confirmed. An idealization was made in such a way that reloading and unloading curves are coincident. It was hypothesized that the stress softening observed within reloading of previously loaded tissue may be described by an evolution of material parameters. These parameters should be related to an alternation of internal structure. The model based on changes in limiting fiber extensibility of fibrillar component of the aortic wall, primarily represented by a collagen, was proposed. The arterial wall was assumed to be hyperelastic transversely isotropic material with different response under primary loading and unloading. A stored energy function was additively split into isotropic and anisotropic part. Preferred direction in continuum, defined in referential configuration, was assumed to be unchanged with cyclic loading. Every straining level in the cyclic test had its own value of fiber extensibility. Explicit form of the relation between evolving limiting extensibility of fibers and maximum previously sustained deformation was proposed in such a way that limiting extensibility under primary loading is considered as the limit. The isotropic matrix response was modeled using Neo-Hooke term with shear modulus values different under primary loading and reloading, however all reloading values were held the same. The predictions of the model described above were in good agreement with observations.**

***Index Terms*—aorta, damage, limiting fiber extensibility, Mullins effect, stress softening.**

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I. Introduction

SIGNIFICANT progress has been made in an area of blood vessels constitutive modeling in last decade. Since 2000 when Holzapfel et al. [1] have published their model of an artery where anisotropy arises from helically arranged bundles of collagenous fibrils, this approach seems to be dominant. Successful applications in computational analyses were reported for example by Cacho et al. [2] in a coronary artery bypass surgery simulation or by Holzapfel et al. [3] in an artery-stent interaction. This model has recently been modified to account for distributed collagen fibrils orientations, Gasser et al. [4].

A strain energy function based on exponential terms originates from Y. C. Fung and his research in seventies of the last century. This mathematical form was many times validated to be able to capture a material nonlinearity of biological tissues. In 2005 Horgan and Saccomandi [5] suggested a model of the strain energy function motivated by the idea of *limiting chain extensibility* which has been successfully used in polymer mechanics; see Gent [6], or Horgan and Saccomandi [7]. In [5] this approach was modified to *limiting fiber extensibility* suitable for composite materials with progressive large strain stiffening. Horny et al. [8] used this model to describe mechanical response of a coronary artery bypass graft and in a constitutive modeling of human aorta [9].

Models mentioned above are capable to describe elastic arterial response. However, it is well known that blood vessels show some inelastic effects [1]. A visco-elastic behavior (dumping, creep, relaxation) can be captured using an internal variables approach [10]. Another inelastic phenomenon in vitro observable in arteries is a stress-softening similar to the *Mullins effect* well known in polymer physics. This kind of strain-induced softening is named after Leonard Mullins due to his extensive research on this topic within the last century [11,12]. The Mullins effect in idealized form is described as follows. When previously non-loaded (so-called virgin) material is loaded to a particular value of the deformation, a stress-strain curve is usually called as *primary loading curve*. Within unloading process the stress-strain curve does not coincide with primary loading and a stress softening is attained. Subsequent loading curve coincide with unloading related to previous cycle. When the value of previous maximum deformation is reached, then the stress-strain curve returns to previous stress maximum and primary loading continues. Described behavior is depicted in Fig. 1.

Except the Mullins effect soft tissues exhibit further softening within so-called *preconditioning* that is defined as deformation process necessary to obtain repeatable mechanical response under cyclic loading.

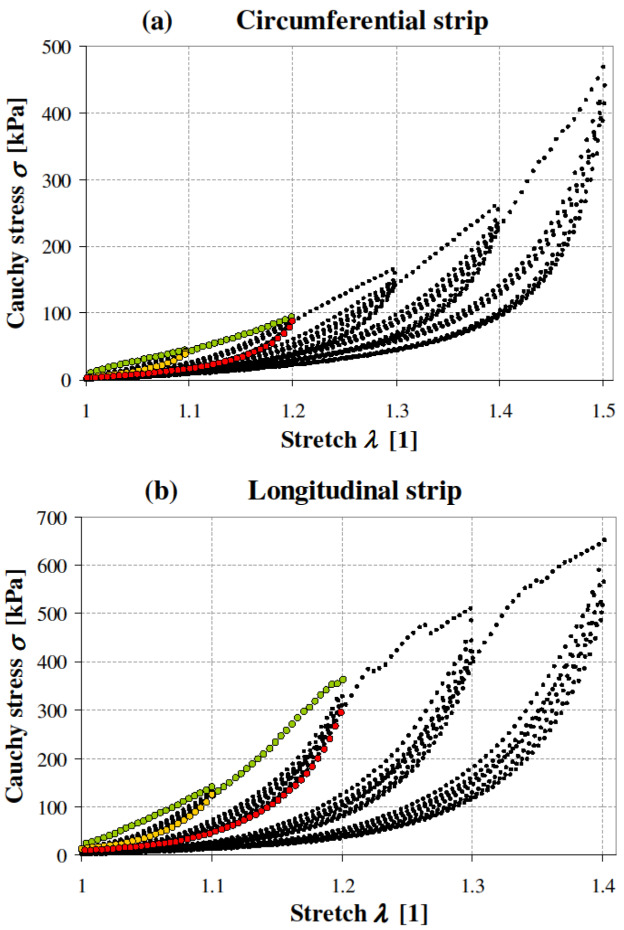

Fig. 1 Stress-strain curves recorded for cyclic uniaxial tension of the human thoracic aorta. Each level of maximum stretch (λ=1.1;1.2;1.3;1.4 and 1.5) was cycled four times. Unloading and reloading curves do not coincide exactly and significant hysteresis remains after four cycles. However, maximum of the stress-softening phenomenon is realized between primary loading and the first unloading. Colored points correspond to cycles used within regression analysis – green circles - primary loading; yellow circles - λ=1.1 fourth unloading; red circles - λ=1.2 fourth unloading.

The Mullins effect in elastomeric materials is also related with a presence of residual strains and induced anisotropy, see Diani et al. [13,14].

Many attempts were made to find suitable constitutive description of the Mullins effect [13]. However, the best choice is still in question. There are two main theories in use. The first approach is based on *Continuum Damage Mechanics* which operates with a damage parameter considered as an internal variable. The method was proposed by Simo [15] and applied for example by Guo and Sluys [16] or Gracia et al. [17]. Ogden and Roxburgh [18] proposed *pseudo-elastic* model to describe the Mullins effect. This was further modified with Dorfmann and Ogden [19] to capture residual strains. Both continuum damage theory and theory of pseudo-elasticity are similar, however the first explicitly employs Clausius-Planck inequality.

Above mentioned models are purely phenomenological. Second main approach is physically motivated and several considerations for internal structure of a material are being made [13]. The leading is so called *network alternation theory*, see e.g. Marckmann et al. [20]. Similar idea may be traced back to Mullins and Tobin [12] who supposed that rubber is a two phase continuum in which stiff phase is transformed to compliant one depending on a deformation. Modern network alternation incorporates knowledge about macromolecular structure of a rubber. The Mullins effect is considered to be a consequence of a breakage of links and increasing length of chains, Chagnon et al. [21], thus softening is based on the deformation depending network evolution.

The softening effect and pseudo-elastic mechanical response of arteries have been known for a long time [22]. However, only few attempts have been made to develop new theories. Especially in last two years a publication activity has grown. Pena and Doblare [23] used anisotropic pseudo-elastic approach to reproduce the Mullins effect observed in an uniaxial tension of a vena cava. Damage mechanics was employed by Pena et al. [24] in modeling of aortic uniaxial tension. A generalized model based on internal variables applicable to arteries was also proposed by Ehret and Itskov [25]. This model can take account for a preconditioning behavior.

The main goal of this paper is to present hypothesis that the Mullins effect observed in uniaxial tension of an artery can be captured by methods of pseudo-hyperelasticity using limiting fiber extensibility depending on maximum previous deformation. There are supposed two ways of material response both governed by hyperelastic description. The first is primary loading and second is unloading/reloading described with changed limiting fiber extensibility parameter. To account for changed response of an isotropic matrix shear modulus values different under primary loading and reloading are also supposed, however constant value for every unloading/reloading is held. Explicit form of limiting fiber extensibility dependence on maximum previous stretch is suggested.

II. METHODS

Cyclic uniaxial tensile tests with four samples of human thoracic aorta were performed with MTS Mini Bionix testing machine (MTS, Eden Prairie, USA). Samples were obtained from cadaveric donors with the approval of the ethic committee in the University Hospital Na Kralovskych Vinohradech in Prague. All experiments were performed within 48 hours after the death. Two samples were tested in the direction aligned circumferentially with respect to natural configuration of an artery and other two aligned longitudinally. Referential geometries were obtained via digital images analyses performed in NIS-Elements software (Nikon Instruments Inc., Melville, USA).

An extension and loading force were measured by MTS testing machine. The cyclic loading was applied as follows: five levels of maximum deformation were performed according to stretch λ=1.1; 1.2; 1.3; 1.4 and 1.5, where λ is the ratio between current length l and referential length L. Each level was cycled four times. It means that for instance after four cycles limited to λ=1.1 the maximum of the deformation was increased to 1.2 and new four cycles performed. True recorded data are shown in Fig. 1.

Employing incompressibility assumption loading stresses σ were obtained as

$$\sigma = \frac{\lambda F}{S} \tag{1}$$

In (1) F denotes applied force and S is the cross-section in the reference configuration.

III. MODEL

Arteries were modeled as an incompressible hyperelastic continuum in which the anisotropy rises from the reinforcement by one family of fibers. In such a case constitutive equations can be read in the form of (2).

$$\sigma_i = \lambda_i \frac{\partial W}{\partial \lambda_i} - p \quad i = 1,2,3 \tag{2}$$

Here p is the Lagrangean multiplicator determined from boundary conditions. W denotes stored energy function. The model based on limiting fiber extensibility, originally proposed in [5] was incorporated in the form published in [8]. It is expressed in (3).

$$W = \frac{c}{2}\left(I_1 - 3\right) - \frac{\mu J_f}{2} \ln\left(1 - \frac{\left(I_4 - 1\right)^2}{J_f^2}\right) \tag{3}$$

Here I_1 denotes first invariant of right Cauchy-Green strain tensor (4).

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \tag{4}$$

c and μ are stress-like material parameters, and J_f is so-called *limiting fiber extensibility parameter* (dimension-less). Additional strain invariant I_4 is expressed as follows.

$$I_4 = \lambda_f^2 = \lambda_1^2 \cos^2 \beta + \lambda_2^2 \sin^2 \beta \tag{5}$$

It arises from material anisotropy and has the meaning of square of the stretch in preferred (fiber) direction. β denotes declination of fibers relative to circumferential direction in natural configuration of an artery. λ_1 is the stretch ratio of the strip aligned with circumferential direction of an artery (λ_2 is aligned longitudinally). The model (3) is called limiting extensibility model due to the existence of a finite value of a fiber stretch $\lambda_f = \sqrt{(J_f + 1)}$ in which the stored energy approaches infinity. Equation (3) should be combined with the condition $I_4 > 1$ to ensure that fibers, which are modeled via preferred direction in continuum, contribute to the stored energy only under a tension. Shear strains/stresses were not considered in the model.

As mentioned above the crucial hypothesis of this paper is such that the stress-softening can be captured via changing J_f. This is motivated by the approach proposed by Chagnon et al. [21] who modeled Mullins effect in elastomers with the Gent model (isotropic counterpart to limiting fiber extensibility model). They suggested an evolution of limiting chain extensibility parameter (and referential shear modulus) in exponential form. Current value of these parameters was related to maximum previously sustained value of the first invariant of right Cauchy-Green strain tensor.

In order to meet the simplicity of the model and mimicking the main idea of (3), limiting fiber extensibility, here we suggest equation (6) as the model for the material alternation.

$$J_f = J_{f0} \quad \text{if } \lambda_{\max} = 1$$

$$J_f = J_{f0}\left(1 - e^{-k\frac{1}{2}\left(\lambda_{\max}^2 - 1\right)}\right) \quad \text{if } \lambda_{\max} > 1 \tag{6}$$

In (6) J_f is the current value of limiting extensibility parameter. It depends on two positive material parameters J_{f0}, k and maximum tensile stretch sustained within loading history (which was used to initiate unloading), $\lambda_{\max}$. Equation (6) governs the evolution of the fiber extensibility in such a way that maximum admissible value of J_f is J_{f0}. This is in contrast to [21] where no limit for the evolution is considered.

J_{f0} is related to the behavior during primary loading. This value is prescribed directly because of no unloading process, which releases the stress softening, have been performed in primary loading path. In other words it means, because $\lambda_{\max}$ is the maximum value of the stretch in the loading history with realized unloading, that $\lambda_{\max}$=1 for primary loading.

Equation (6) is increasing function (if softening was initiated) of $\lambda_{\max}$. Contrary to (6), where limiting extensibility is considered as increasing function of $\lambda_{\max}$, isotropic shear modulus is modeled as decreasing. It is in accordance with Chagnon et al. [21]. They suggested an exponential equation with two material parameters to describe its softening. Here we do not want to raise number of model parameters and only two values of μ, see (3), are supposed. μ_0 is related to primary loading and its softened value is μ_{12}.

According to (2) σ_1 and σ_2 can be computed (σ_3=0 to determine p). Equation (3) generally involves three stretch ratios, reduced to two assuming material incompressibility (7).

$$\lambda_1 \lambda_1 \lambda_3 = 1 \quad (7)$$

However, only displacements in loading directions were recorded (due to the testing machine design). Thus we incorporated additional boundary condition which restricts the value of the transversal stress to be zero (lateral surfaces of the strip are non-loaded within uniaxial tensile test).

IV. RESULTS

Recorded data engaged in parameters estimation are depicted in Fig. 2. Primary loading curve and the fourth unloading curve for λ=1.1 and λ=1.2 were included in regression analysis (the idealization assumes unloading=reloading). Subsequent material responses were not considered due to non-convexity of primary loading for λ > 1.2 in longitudinal strip (see Fig. 1). This kind of behavior was not repeated in the circumferential strip and may not be materially intrinsic (e.g. possible slippage in clamps). Material parameters were estimated using weighted least square method in Maple (Maplesoft, Wareloo, Canada). They are summarized in Tab. 1. Parameters c and β were held the same for all data. Shear modulus μ_0 was considered for primary loading curve and μ_{12} for unloading curves. The limiting extensibility parameter was being varied with successive loading as described above. Thus, used material parameters can be summarized this way: primary loading parameters – c, μ_0, J_{f0} and β; all unloading/reloading – c, μ_{12}, J_{f0}, k and β. Results are displayed graphically in Fig. 2.

Total number of used data points was 81. The numerical procedure satisfied the condition of tensile loading in the preferred direction (I_4 > 1) in all points except two. Boundary conditions used to determine lateral deformation were satisfied properly everywhere (values did not exceed 7% of magnitude of loading stress).

Table 1 Material parameters

c	μ_0	μ_{12}	J_{f0}	k	β
[kPa]	[kPa]	[kPa]	[1]	[1]	[°]
85.64	3000	5	0.120	1.515	49.2

V. CONCLUSIONS

All experiments proved the presence of the Mullins effect within cyclic loading of human thoracic aorta. Two perpendicular strips were engaged in the regression analysis. It was found that the stored energy density function (3) based on limiting fiber extensibility is capable to describe stress-strain curves obtained from cyclic uniaxial tensile test. Varying two parameters (μ and J_f) was sufficient for the description of the stress softening like the Mullins effect. It was suggested that changes in these parameters were strain-induced, probably due to an alternation in internal structure of the material.

Explicit form of the limiting fiber extensibility evolution based on exponential function was suggested (6). The functional dependence mimics the idea of limiting extensibility. To meet simplicity and do not rise errors in the model (lateral deformation was not measured but determined from boundary condition) evolving J_f is considered as function of maximum previously sustained deformation in the loading direction. One may expect that relating current value of J_f to maximum stretch in the fiber direction could be more appropriate; however results show that proposed description fits experimental data well.

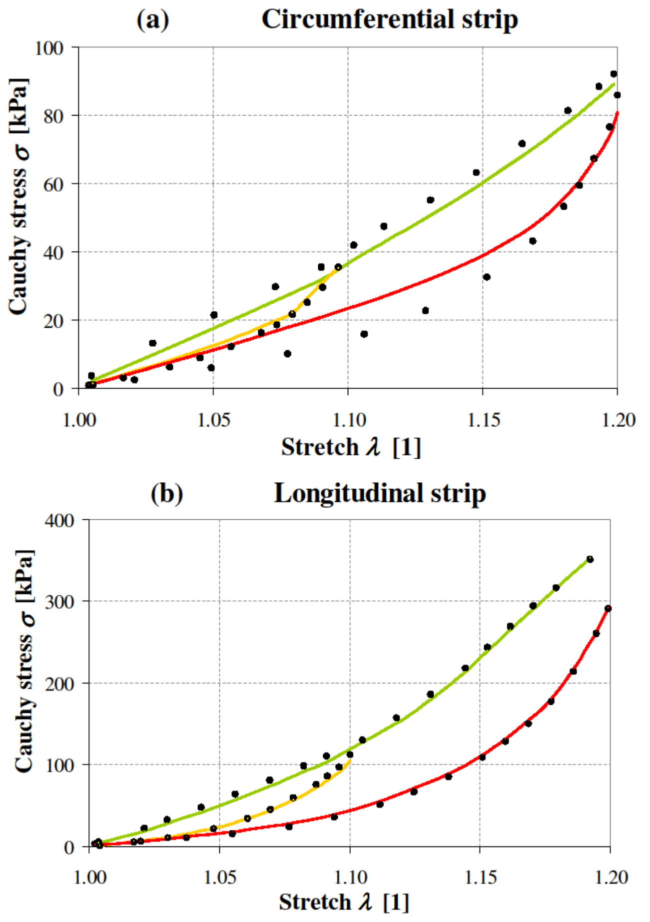

Fig. 2 Typical stress-strain curves and model fitting. Only points selected for the regression analysis are presented.

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