# Model of Behavior at Pulling of the Reinforced Concrete of Waved Fibers

Bouzeroura. Mansour, Bouafia. Youcef, Kachi. Mohand said

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*Abstract*—**The modeling of the behavior in pulling of the reinforced concrete of waved fibers (to variable geometry), is based on the study of the behavior of the fiber in the concrete matrix. Writing the energy equilibrium of an undulating fiber at the time of the sliding, the movement is governed by a differential equation. The pull-off strength of the fiber is obtained by integrating this one on the length anchored of the fiber, the phenomenon of friction fiber-concrete matrix, the influence of the radial constraints (due to the undulation) and the case of the possible plasticization of the steel constituting the fiber, one considered in the modeling. A solution to the problem of the modeling of the adhesion waved fiber - concrete matrix was proposed bringing to light the evolution of the constraint of adhesion ($\tau_u$) according to the "t" anchoring. This modeling is introduced in an effort relation-opening of the cracks, and then a passage to the law (constraint –deformation) was made by modifying this relation. Finally for every modeled parameter (adhesion, constrained-deformation in pulling and pickets submitted to a compound flexion), a calculation confrontation /test was been executed. The results of the confrontation are rather satisfactory.**

*Index Terms*— **adhesive, concrete fiber, fibers corrugated (waved), friction, modeling**

## I. INTRODUCTION

Behavior of the composite (concrete fibers) is elastic linear until cracking of the matrix. The interior studies that one can quote [1], [2] and [5] show that at the instant where appears a crack there are a brutal decrease of the constraint.

We interest us in that follows to the behavior after cracking, therefore after the peak of load. In this domain, one considers the sliding phenomenon of fiber-matrix eventual plasticization of the fiber steel. One expresses then the energizing equilibrium of a fiber at the time of the sliding.

Bouzeroura. Mansour. Is with the University of Abdderahmane Mira, Targa Ouzemour street 06000 Bejaia, Algeria, and with (Materials and structures modeling laboratory), of Tizi-Ouzou University. (Corresponding author to provide phone: 00213-555-251671; fax: 00213-21-5105; (e-mail: Bouzeroura2001@yahoo.fr).

Bouafia. Youcef. Is the head of LaMoMS laboratory, University of Tizi-Ouzou Hasnaoua II, Bt C 15000 Algeria (e-mail: Bouafia_y@yahoo.fr).

Kachi. Mohand said.  Is with the University of Tizi-Ouzou Hasnaoua II, Bt C 15000 Algeria (e-mail: Kachi_ms@yahoo.fr).

## II. MODELING OF THE ADHESION MATRIX IN CONCRETE CAPTION

### A. *Equation of static equilibrium for an element of wire-drawn fiber*

The wire-drawn fiber (said also to variable geometry) is decomposed in curves sections of curvilinear elementary length "ds", Fig. 1. The equilibrium can be written in the following manner, Eq. (1), (2), (3) and (4). The projection of the forces along;

The x axis:
$$P\sin(\frac{d\theta}{2})+(P+dP)\sin(\frac{d\theta}{2})=dN \tag{1}$$

The z axis:
$$dP\cos(\frac{d\theta}{2})=dT \tag{2}$$

For the small angle to elementary elements "ds", we can do:

$$\sin(\frac{d\theta}{2})=\frac{d\theta}{2} \text{ , and } \cos(\frac{d\theta}{2})=1$$

One ignoring the higher order terms $dP(\frac{d\theta}{2})$, the Eq (1) and Eq (2) become Eq (3) and (4). [5]

$$dN = Pd\theta \tag{3}$$
$$dP = dT \tag{4}$$

The equilibrium of an element of fiber at the time of the sliding, generated a radial effort " $dN$ " and the tangential effort " $dT$ ", that two efforts can be linked by a friction law of coulomb, this law consists to linking the shear stress "$\tau$" to the normal stress "$\sigma$" of a facet by a friction coefficient " $f$ " and a cohesion "$C_o$". We will adopt a coulomb's law in global forces, where the Eq (4) as Eq (5).

$$dT = f \, dN + \tau_o \, p \, ds \tag{5}$$

Where " $p$ " is the perimeter of the fiber and " $\tau_o$ " is the bond stress, between the fiber and the matrix.
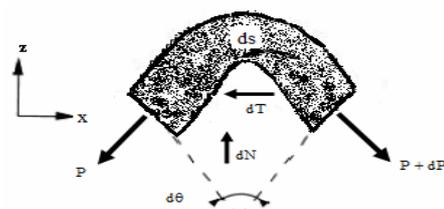


Fig. 1: Equilibrium of small element wire-drawn fiber [5]

*B. Energy balance sheet during sliding*

Physically at the time of the sliding, a fiber element undergoes, on the one part, a sliding of rigid body of amplitude "$d\delta$" and on the other part, a variation of curvature "$dC$" produced by flexion. The fiber follows then the print of its initial geometry, the experimental observances of [5] showed that each section undergoes a plasticization. The purely statics approach of the fiber equilibrium does not permit an assessment of plastic energy dissipated during the sliding of the fiber. The equilibrium of this fiber element curve must be described from an energy point of view.

The balance of the mechanical energy, applied to the slipping elementary length curve of a fiber that, allows writing that the work of the external forces "$W_{ext}$" is equal to the work of the deformations "$W_{def}$".

$$W_{ext} = W_{def} \tag{6}$$

The work of the external forces written as Eq. (7):

$$W_{ext} = \int_S p_i \, u_i \, dS \tag{7}$$

Correspond to the integration, of the product of the elementary forces "$p_i \, dS$", over the external surface "$S$" of the fiber elements, "$p_i$" is the pressure exerted on the exterior surface "$u_i$" are the kinematic displacement field of the element , "$_i$" is the indication by representing a coordinate in the space. The strengths of mass, such as the gravity, are disregarded.

In the field of the displacements considered, only the components of the efforts ($P$, $P+dP$ and $dT$) hard working follow the curvilinear axis is taking in account Fig. 1. Disregarding the terms of higher order, the Eq. (7), written as Eq (8):

$$W_{ext} = -P\,d\delta + (P+dP)d\delta - dT\,d\delta$$
$$W_{ext} = dP\,d\delta - dT\,d\delta \tag{8}$$

The work of the internal efforts is obtained by integration, over the element's volume "V", the product of the stress tensor "$\sigma_{ij}$" and strain tensor "$\varepsilon_{ij}$" defined in all dawned of the element's volume Eq. (9).

$$W_{def} = \int_V \sigma_{ij} \, \varepsilon_{ij} \, dV = \int_V \sigma \, z \, dC \, dV = dC \int_V \sigma \, z \, dV$$
$$W_{def} = dC \, ds \int_S z \, \sigma \, dS = dC \, ds \, M \tag{9}$$

With:

$M$ : bending moment in the fiber,

$ds$ : Elementary curvilinear length of the fiber.

$dS$ : Straight section of the element's fiber.

Considering the Eq. (8) and (9), the balance of the mechanical energy can be written in Eq. (10):

$$dP \, d\delta - dT \, d\delta = M \, dC \, ds \tag{10}$$

Combining the Eq. (3), (5) and (10), we obtain the Eq. (11).

$$dP = (P f C + \tau_o p + M C') \, ds \tag{11}$$

Where "$C'$" is the first derivative of the curvature "$C$" against the curvilinear abscissa. The result of the integration of the Eq. (11) along the fiber gives a differential equation of the first order in "$P$". It should be noted that the Eq. (1) to (12) were developed by Chanvillard [5].

In order to define the behavior's problem of the wire-drawn fiber in the concrete matrix, the modeling of the fiber adhesion stamps (coefficient of rubbing) and the effect of the tilting of these fibers will do one of the principal aims of this study.

*C. Modeling of the adhesion (coefficient of rubbing)*

The coefficient "$\mu$" is then defined starting from the isostatic equilibrium equations between the asperities in interaction, Fig. 2. Roughness rubbing has a sinusoidal profile, it results the following expressions.

$$\mu = (f + \chi)/(1 - f\chi) \tag{12}$$

$$\chi = \pi \frac{A}{\lambda} \cos\left[\pi \frac{\delta}{\lambda}\right] \tag{13}$$

With;

$\lambda/2$ : Period,

$\chi$ : The slope of the contact,

$A$ : Amplitude of the rough,

$\delta$ : Relative displacement of the rough and $\dfrac{A}{\lambda} \le 0{,}015$ , in [7].

In Eq. (13), the condition $\delta \, \pi \, \lambda/2$ , drive at:

$$\left[\pi \frac{\delta}{\lambda}\right] \pi \, \pi/2 \Rightarrow \chi = 0$$

Where;

$$\left[\frac{\delta}{\lambda}\right] \pi \, 1/2$$

Equations (12), (13) make it possible to plot the curve, Fig. 3, representing the evolution of "$\mu$" according to the ratio "$\dfrac{\delta}{\lambda}$". This one makes it possible to fix the coefficient of apparent friction "$\mu$".

Consequently, the variation of the tangential stress "$T$" given by the Eq. (5) is transformed into Eq. (14).

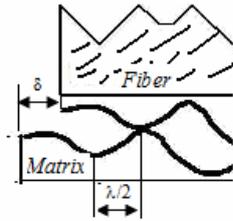$$dT = \mu \, dN + \tau_o \, p \, ds \tag{14}$$
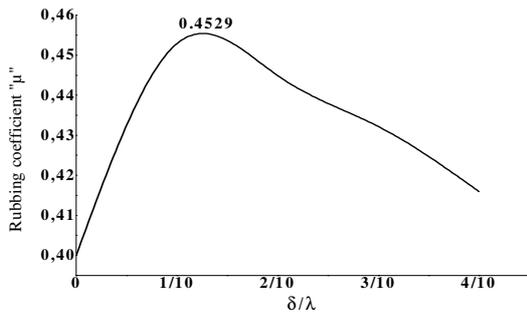
Fig. 2: Sinusoidal profile of the roughness rubbing



Fig. 3: Evolution of the visible rubbing coefficient $\mu$
According to $\delta / \lambda$

*D. Determination of the probabilistic functions density for the case of orientation 2D of fibers in the matrix*

The fiber is tilted of an angle "$\theta$" compared to the axis of traction "$x_1$", Fig .4. This slope causes an effect of polished of friction and consequently, an increase in the forces compared to those of a fiber of null slope [3].

By referring to [7], the expression of the function of distribution " $G$ " is given by the Eq. (15).

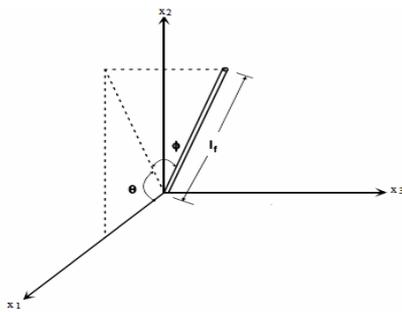$$dG = (\phi,\psi)d\phi d\psi \qquad (15)$$



Fig. 4: Representation of an axis of waved fiber in the space
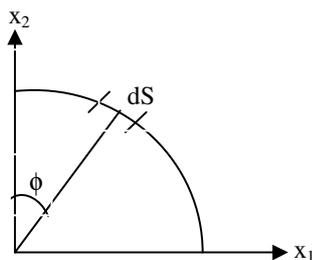


Fig. 5: Definition Space of the probabilistic density function (2D)

The fibers are directed arbitrary in the plan of the matrix concrete, in is interested only in the fibers which are in the plans of the force of pulling, in this case of orientation in 2Dimension, Fig .5, with " $\psi=0$", the expression of " $g(\phi,\psi)$ " is transformed into " $g(\phi)$ ".

The determination of $g(\phi)$, into fixed a space of definition "$S$", represented by a quarter of circle of ray unit. Probability that a point of "$S$" determined by the angle of orientation "$\phi$" is on the part of "$S$", noted "$\Delta S$"is:

$$P_{rob}\left[g(\phi,\psi)\in \Delta S\right] = \frac{\Delta S}{S} = \int_{\Delta S} dG \qquad (16)$$

$$\int_{\Delta S} dG = \frac{\Delta S}{S} = dG \text{ , With}$$

$$dG = (\phi,\psi)d\phi d\psi \quad \text{and} \quad dG = (\phi)d\phi = \frac{dS}{S} = \frac{1.d\phi}{\dfrac{2\pi.1}{4}}$$

$$(\phi)d\phi = \frac{2}{\pi}d\phi \text{ , where } g(\phi) = \frac{2}{\pi} \qquad (17)$$

With 1: represents the unity ray.

The multiplicative factor is " $e^{\mu\theta}$ ", with " $\mu$ " coefficient of rubbing for the couple fiber matrix.

$$P_\theta = e^{\mu\theta} P_{\theta-0} \qquad (18)$$

In axes system ($x_1$, $x_2$, $x_3$), one can write; $\cos\theta = \sin\phi\cos\psi$
And we have Eq. (19);

$$e^{\mu\theta} = e^{\mu\theta ar\cos[\sin\phi\cos\psi]} \qquad (19)$$

The generalization of this expression to all fibers is done by application of the operator mathematical expectation of the function " $g(\phi)$ " given in Eq. (17).

$$\eta = <e^{\mu\theta}> = <e^{\mu\theta ar\cos[\sin\phi\cos\psi]}>$$

For a two-dimensional and random distribution, one has;

$$g(\phi) = \frac{2}{\pi}$$

$$\eta = \frac{2\ e^{\mu\frac{\pi}{2}}}{\mu\ \pi}(1 - e^{-\mu\frac{\pi}{2}}) \qquad (20)$$

The influence of tilted fibers is then taken into account by the parameter "$\eta$", Eq. (21).

Finally, the expression of the effort " $P(s)$ " necessary to pull the wire-drawn fiber of the matrix concrete, is given by the combination of the Eq. (10), (14) and (20), the Eq. (21) obtains some after integration over the length of the fiber [4].

$$P(s) = \eta\frac{\tau_o p + MC'}{-\mu C}\left(e^{\mu Cs} - 1\right) \qquad (21)$$

### III. CONFRONTATION FORCES DISPLACEMENTS CURVE TO EXPERIMENTAL RESULTS

The developed model makes it possible to describe the behavior of a fiber undulated during its wrenching of a concrete matrix. It makes it possible to plot the curve effort of wrenching according to displacement. The confrontation of the curves obtained by the model suggested and the curves obtained on tests of wrenching carried out by [5], for various lengths of fiber ( $\lambda_f$ = 16 and 24 mm), is given to the (Fig .6 and Fig.7). The result of this confrontation is rather satisfactory. The shape of the experimental curves is correctly approximate. The model suggested is based on the application of the theorem of mechanical conservation of energy. The behavior is described using a first order differential equation. It utilizes friction with the interface matrix fiber of concrete, the radial stress modifying those of shearing and the plasticization of steel. These phenomena are brought into play during the wrenching of corrugated fiber. The geometry of fiber plays a dominating part in a test of wrenching. The fiber dissipates, during separation, an energy of friction in contact with the matrix (variable according to the intensity of the normal stress) and, at the time of the slip, an energy of plasticization result to the rectification of fiber. These two phenomena are taken into account. The behavior of the test-tubes is approximate in a rather satisfactory way.
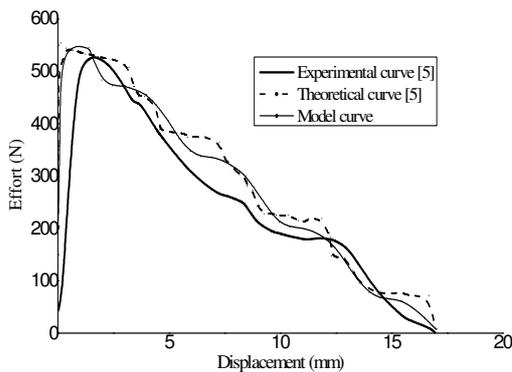


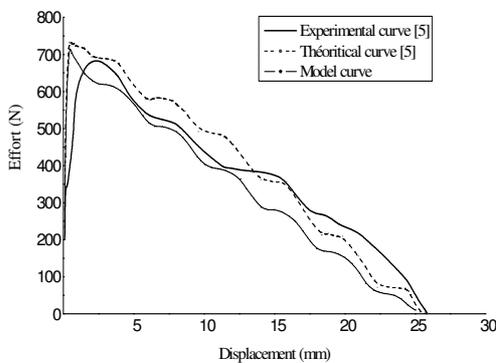Fig. 6: Curve force – displacement ( $\lambda_f$ = 16 mm)



Fig. 7: Curve force – displacement ( $\lambda_f$ = 24 mm)

### IV. ADAPTATION OF THE RELATIONS STRESS DEFORMATION SUGGESTED BY [1] AND [2] TO WIRE-DRAWN FIBER

#### A. Determination of the adhesion constraint $\tau_u$ of the undulating fibers

In order to determine an average constraint of adhesion matrix fiber, the calculation is executed for the three lengths of corrugated fibers quoted previously (8, 16 and 24 mm). This constraint is given by the expression (22).

$$\tau_u = \frac{P(s)}{\pi \phi \, la} \qquad (22)$$

With;

$\phi$ ; Diameter of the fiber,

*La*; adherent length of the fiber (*la* = $\lambda_f$ . t /2),

t; parameter of anchorage [1, 2].

$P(s)$ ; Effort of wrenching fiber,

The term "t" was the subject of a parametric study. The evolution of the constraint " $\tau_u$ " according to anchoring "t" is represented to the Fig. (8), for the three lengths.
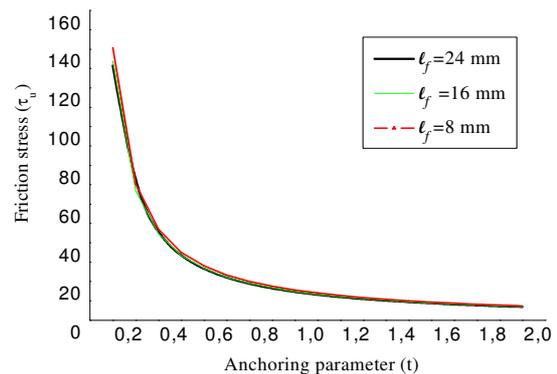


Fig. 8: The evolution of $\tau_u$ according to the parameter of anchorage t.

The value of $\tau_u$ adopted corresponds to the lower limit of the curve (with the stabilization of this one). When the value of "t" becomes higher than 1.4 and until to 2, the constraint of adhesion " $\tau_u$ "does not evolve almost any more. The three curves converge towards the same value ( $\tau_u$ = 7.92 MPa).

This last value is then introduced into the relations stress-strains (23, 24) suggested by [1] and [2].

These relations are then modified for better taking into account the undulating fibers. From the expressions of opening of the cracks, the authors then made a passage to stress-strains relations; these are given by Eq. (23).

$$\begin{cases} \sigma = E_{ct}\,\varepsilon & si \quad 0 \le \varepsilon \le \varepsilon_{ft} \\ \sigma = \sigma_{uc} - [\sigma_{uc} - f_{ft}\,]\dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_{ft} - \varepsilon_u)^6} \\ & si \quad \varepsilon_{ft} \le \varepsilon \le \varepsilon_u \\ \sigma = \sigma_{uc}\,[\,1 - \dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_{rt} - \varepsilon_u)^6}\,] & si \quad \varepsilon_u \le \varepsilon \le \varepsilon_{rt} \end{cases} \tag{23}$$

$$\begin{cases} \sigma = E_{ct}\,\varepsilon & si \quad 0 \le \varepsilon \le \varepsilon_{ft} \\ \sigma = \sigma_{uc} - [\sigma_{uc} - f_{ft}\,]\dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_{ft} - \varepsilon_u)^6} \\ & si \quad \varepsilon_{ft} \le \varepsilon \le \varepsilon_u \\ \sigma = \alpha\sigma_{uc}\,[\,1 - \dfrac{(\varepsilon - \varepsilon_u)^\beta}{(\varepsilon_{rt} - \varepsilon_u)^\beta}\,] & si \quad \varepsilon_u \le \varepsilon \le \varepsilon_{rt} \end{cases} \tag{24}$$

The effort of pulling and the shear stress are given respectively by the Eq. (21) and (22). According to these considerations the Eq. (23) are transformed into Eq. (24). They thus make it possible to take account of the corrugated geometrical shape of fiber. [3].

With;

$\sigma_{uc}$ ; maximum ultimate stress,

$\varepsilon_{ft}$ ; deformation of the concrete cracking,

$\varepsilon_{rt}$ ; rupture of the composite tensile shear,

$f_{ft}$ ; tensile strength of the composite,

$E_{ct}$ ; initial module of composite tensile,

$\varepsilon_u$ ; strain corresponding to the total mobilization, of the fiber matrix adhesion.

$\alpha = 0,8$ , $\beta = 7$ .

The parameters "$\alpha$" and "$\beta$" are given starting from the confrontation of the results Eq. (24) to experimental results. The model suggested is introduced into nonlinear computation software, until rupture, of a section of beam "CMP". [6].

*B Confrontation of the relation stress-strain in direct pulling*

These relationships comparison is made to the results obtained on cylindrical specimens tested at direct tensile by Zhan [8], these specimens of metallic fibers wavy reinforced concrete are BFO25 and BFO40 respectively determined to 25 kg/m³ and 40 kg/m³. The mechanical characteristics of concrete and fibers are given in tables (1 and 2).

Tab. 1: Mechanical characteristics of the composite
(Concrete reinforced with steel fibers)

| $f_{ci}$ | $f_{ti}$ | $R_c$ | $\varepsilon_{rt}$ | $\varepsilon_{cu}$ |
|---|---|---|---|---|
| (MPa) | | | " ‰ " | ‰ " |
| 47,7 | 2,94 | 0,7 | -50 | 3,5 |

Tab. 2: Characteristics of the fibers

| $\tau_u$ | $E_f$ | $\lambda_f$ | $\phi$ | $\varpi$ | $\varepsilon_u$ |
|---|---|---|---|---|---|
| "MPa" | "GPa" | "mm" | "mm" | "%" | " ‰ " |
| 7,92 | 200 | 60 | 1 | 0,31 | - 0,74 |

With; $f_{ci}$ ; compressive resistance of the composite,

$f_{ti}$ ; tensile strength of the composite,

$R_c$ ; ductility material coefficient

$\varepsilon_{cu}$ ; concrete compressive breaking strain,

$E_f$  ; Elastic fiber module,

$\varpi$ ; Percentage by volume of fiber,

The specimens Strains tested have been obtained by dividing the crack measured by the length of the extensometers, fixed on the specimen (140 mm). The comparison of the results is illustrated in Fig. 11, and detailed in Fig. (12).
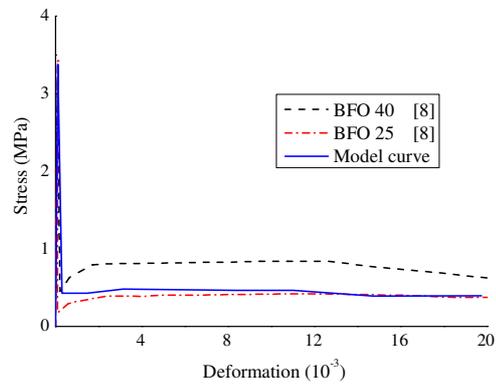


Fig. 09: Comparison with the results obtained by [8]
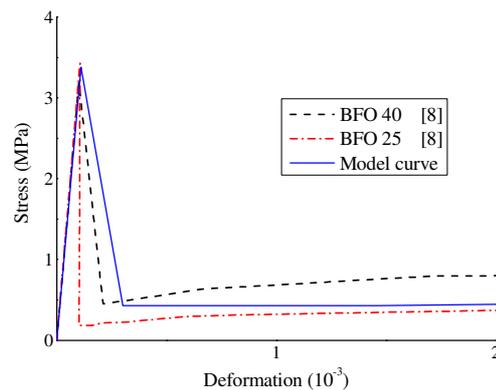(Fiber length lf = 60 mm)



Fig. 10: Detail of figure 11

According these results, there is the phase before the break is elastic linear, the tensile strength is substantially the same between the two test pieces and the proposed model, then there

is a brutal fall effort which looks after the rupture of the composite and a residual strength maintains in the finals phase. Finally to a strain of approximately $10^{-2}$, the load tends to decrease. It can be concluded that the behavior is approached in a satisfactory manner

## V. CONCLUSION

The incorporation of steel fibers in a matrix concrete gets for the composite a ductile behavior in traction. By effect of sewing of the cracks, the fibers limit their spread and absorb a certain quantity of energy.

This phenomenon a function of the geometry and the mechanical characteristics of fiber used. In order to determine the contribution of fibers well to the behavior of the composite in traction, it is essential to take account of the shape of fibers (right, with or without anchoring at the ends or corrugated). Within the framework of this work, one was interested in corrugated fiber. Modeling is based on the effect of rubbing and friction and takes into account the undulations of fiber.

A parametric study made it possible to model the friction of fiber undulated in a concrete matrix and to highlight the evolution of the friction stress " $\tau_u$ "according to the parameter of anchoring "t". This modeling is introduced by adaptation of the relations stress-strains suggested for a concrete reinforced with fibers in traction by [1] and [2].

The software, making it possible to follow the nonlinear behavior until the rupture of a concrete section, proposed by [7], is used to validate this modeling. The results of confrontations carried out are satisfactory.

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